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EFFECT OF OVERTAKING DISTURBANCE ON THE MOTION OF CYLINDRICAL HYDROMAGNETIC STRONG SHOCK WAVES IN A SELF GRAVITATING GAS.

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ABSTRACT

The chisnell' chester[2]-whitham[3] method has been used to study the propagation of diverging cylindrical hydromagnetic shock waves through a self gravitating gas in the presence of a magnetic field having only constant axial(Hz_0) component, the problem is investigated for strong shock only. Assuming an initial density distribution p„=p'r", where p' is the density at the axis of symmerty and a constant, the analytical relation for shock velocity and the shock strength pressure, particle velocity and density have been obtained.

INTRODUCTION

The study of propagation of cylindrical hydromagnetic shock waves through a self gravitating gas is relevant to the phenomenon of thunder (Kumar and Prakash)[45]. Chaturani[6] has obtained similarity solution decribing the propagation of diverging strong cylindrical hydromagnetic shock waves. In the present paper the CCW method has been used to investigate this problem for strong shock. Assuming an initial density distribution, p^p'r", therelation for shock velocity and shock strength, pressure, particle velocity and density have been obtained for strong shock. Case of strong shock is explored under two district situations, viz (i) when the ratio of densities an either side of the shock nearly equal (y+l)/(r-i), where *y* is the adiabatic index of the Gas or (ii) when the applied magnetic field is strong. Basic Equations :-

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial r}+\frac{1}{p}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial t}+\frac{\partial u}{\partial t}p\frac{\partial H^2}{\partial t}=0$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+p\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)=0 \qquad (1)$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+a^2\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)=0$$

$$\frac{\partial m}{\partial r}-2\pi r p=0$$

$$\frac{\partial H}{\partial t}+u\frac{\partial H}{\partial r}+H\frac{\partial u}{\partial r}+H\frac{u}{r}=0$$

Where u,v, p, p„ H are respectively, the radial and azimuthal velocity, density, constant permeabily and axial magnetic field of the gas made notes the mass inside the Glinder of radivs r and unit length and

$$a^2=\gamma\ p/p$$

3. Boundary Conditions :- The magneto hydro-" dynamic conditions can be wirtten in terms of a single parameter % = p/p_0as

$$p=p_0\xi,\ \mathrm{H}=H_0\xi,\ \mathrm{u}=\frac{\xi-1}{\xi}U$$

$$U^2=\frac{2\xi}{(\gamma+1)-(\gamma-1)\xi}\left[{a_0}^2+{b_0}^2\{(2-\gamma)\xi+\gamma\}\right]$$

$$\mathrm{p}=p_0+\frac{2p0(\xi-1)}{(\gamma+1)-(\gamma-1)\xi}\left[{a_0}^2+\frac{\gamma-1}{4}{b_0}^2(\xi-1)^2\right] \qquad (2)$$

(2)

Where 0 and 1, respectively stand for the state immediately ahead and behind the shock front, U is the shock velocity a^0 is the sound speed $(\gamma P_o/P_o)^1/^2$ and b_0 Alfeven speed $(\mu H_0^2/p_0)^1/^2$

Strong Shock-In the limiting case of a strong shock p/p_0 is large. In the magnetic case this achieved in two ways.

Case I : The purely non-magnetic way when the ratio of density on either side of the shock nearly equals γ+l/γ -1 or

Case II: When the applied magnetic field is large i.e. b_0^2 »a_0^2 or when μH_0^2»γp_0 i.e. when the ambient magnetic pressure is very much of greater than the pressure in equilibriun state in term of % ξ the boundary condition become

$$\rho = \rho_0 \xi, \quad H = H_0 \xi, \quad u = \frac{\xi - 1}{\xi} U, \quad \frac{p}{p_0} = \chi(\xi)\left(\frac{U}{a_0}\right)^2 + L,$$

$$\chi(\xi) = \frac{\gamma(\gamma-1)(\xi-1)^3}{2\xi[(2-\gamma)\xi+\gamma]} \quad \text{and} \quad L = \frac{\xi(\gamma+1)-(\gamma-1)}{(\gamma+1)-(\gamma-1)\xi} \tag{3}$$

Characteristic Equations

The Characteristic form of the system of equation (i) is easily obtained by forming a linear combination of first and third equations of the system of equations (i) in any one direction in (r,t) plane, can be written as

$$dp + \mu H dH + \rho c du + \rho c \frac{Gm}{(u+c)^2}\frac{dr}{r^2} + \rho c^2 \frac{u}{u+c}\frac{dr}{r} = 0 \tag{4}$$

In order to estimate the strength of overtaking distrubances independent C_t characteristic is considered.

The differential equation valid across a C. distrubances is written by replacing Cy-C in equation (4) we get,

$$dp + \mu H dH - \rho c du - \rho c \frac{Gm}{(u+c)^2}\frac{dr}{r^2} + \rho c^2 \frac{u}{u-c}\frac{dr}{r} = 0 \tag{5}$$

Analytical Relation for flow variables

Assuming, the initial density distribution of the form

$$P_o = P^1 r^{-w} \tag{6}$$

The equilibrium state of a gas is assumed to be specified by the condition

∂/∂ =0=u and H_{z0}= constant

Using (7) the first equation of the system of equation (1) the hydrostaic equilibrium condition previling in fornt of the shock can be written as

$$\frac{1}{\rho_0}\frac{dp_0}{dr} + \frac{Gm}{r^2} = 0 \tag{8}$$

The intergation of equations (8) we get

$$p_0 = K^1 - \frac{G\rho^2 2\pi r^{1-2w}}{(1-2w)(2-w)} \tag{9}$$

$$\frac{p_0}{p^1} = K + K_1 r^{1-2w} \tag{10}$$

where $\left(K = \frac{K^1}{P'^2}\right)$

$$K_1 = \frac{2\pi\gamma}{\gamma(1-2W)(w-2)}$$

Where $D = a^{1/2} / G\rho^1$

Now Substituting the shock condition (3) into (4) we get

$$\frac{dU^2}{dr} + \frac{B_2}{r}U^2 + B_2 \frac{G 2\pi \rho^1}{(2-w)} r^{-w} = 0 \tag{11}$$

Where $B_1 = \frac{B_1^1}{A} \quad B_2 = \frac{B_2^1}{A}$

$$B_1^1 = \left\{ \frac{\chi(\xi)(\xi-1)}{(\xi-1)+[\chi(\xi)]^{1/2}} - \frac{\chi(\xi)w}{\gamma} \right\}$$

$$B_2^1 = \left\{ \frac{\xi\sqrt{\chi(\xi)}}{(\xi-1)+\sqrt{\chi\xi(\xi)}} - L \right\}$$

$$A = \left\{ \frac{\chi(\xi)}{\gamma} + \frac{\xi-1}{2}\sqrt{\frac{\chi(\xi)}{\xi}} \right\}$$

On integration (11 we get) we get

$$U^2 = T^1 r^{-B} - \frac{2\pi B_2}{(2-w)} \frac{a'^2}{D} . \frac{r^{1-w}}{(1+B_1-w)} \tag{12}$$

Where T^1 is the constant of intergation. It describes free propagation.

For the C-disturbance generated by the shock the fluid velocity increment using (11) into (3) may be expressed as

$$du_- = \frac{\xi-1}{\xi} . \frac{1}{2U} \left\{ -B_1 U^2 \frac{dr}{r} - \frac{B_2 2\pi G \rho^1}{(2-w)} r^{-w} dr \right\}$$

on substituing the shock condition (3) into (5) and (8) we get

$$A * \frac{dU^2}{dr} + B_1^{*} \frac{U^2}{r} - B_2^{*} \frac{G 2\pi \rho^1}{(2-W)} r^{-w} = 0 \tag{14}$$

Where

$$A^* = \left\{ \frac{\chi(\xi)}{\xi} - \frac{1}{2}(\xi-1)\sqrt{\frac{\chi(\xi)}{\xi}} \right\}$$

Table-1: EOD on the propagat ion of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables : Taking p_0/p_1 = 0.9 at γ = 0.1, γ = 1.4, D = 20.0 w = 1, U/a_0 = 20 at r - 0.1, ξ = 1.5, K_1 = 0.439600, K = -3.49600, A = 0.027806, L = 1.77778, $\chi(\xi)$ □0.010145, B_1 = 0.032064, B_2 = -53.259668, A* = -0.013313, B_1* *m* -0.467311, B_2* = 155.834225.T = -49.902389.T = -541.984730

r			U/a_0			U/a^1			P/P'			P/P'		u/a'	
	p_0/p_1	FP	EOD	CP%	FP	E00	CP%	FP	EOD	CP%	FP	EOD	FP	EOD	CP%
0.099546	0.920049	20.084792	20.114645	0.14	6.078336	6.087362	0.14	5.400928	5.412188	0.20	15.068411	15.068411	2.029112	2.029121	0.14
0.099656	0.915174	20.064177	20.114645	0.11	6.059334	6.066149	0.11	5.364626	5.373038	0.15	15051778	15.051778	2.019778	2.022050	0.11
0.099765	0.910355	20.043.794	20.059187	0.07	6.040698	6.045160	0.07	5.328828	5.334529	0.10	15.035333	15.035333	2.013566	2.015053	0.07
0.905502	0.023270	20.031445	20.005400	0.04	6.021547	6.024006	0.04	5.292866	5.295874	0.05	15.018733	15.018773	2.007182	2.008002	0.07
0.099984	0.900703	20.002978	20.00402	0.005	6.002756	6.003069	0.005	5.257393	5.257774	0.00	15.002400	15.002400	2.000919	2.001023	0.01

Table-2: EOD on the propagat ion of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables : Taking p_0/p_1 = 0.9 at γ= 0.1, γ - 1.4, D = 20.02 w =1, U/a_0= 20 at r = 0.1, ξ= 1.5, K_1 = 0.439161, K = -3.491608, A = 0.027806, L = 1.77778, $\chi(\xi)$ = 0.010145, B_1 = 0.032064, B_2 = -53.259668, A*= -O.013313, B_1* = -0.467311, B_2* = 155.834255, T = -369. 915858, T = -53.25968

r			U/a_0			U/a'			P/P^1			P/P^1		u/a^1	
	p_0/p_1	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	FP	EOD	CP%
0.099546	0.920031	20.084764	20.027372	-0.28	6.078268	6.060900	-0.28	5.400812	5.379325	-0.39	15.068411	15.068411	2.026089	2.020300	-0.28
0.099656	0.915161	20.064147	20.020730	-0.21	6.059282	6.046170	-0.21	5.364539	5.348381	-0.30	15.051778	15.051778	2.019761	2.015390	-0.21
0.099765	0.910347	20.043763	20.014154	-0.14	6.040485	6.031562	-0.14	5.328769	5.317815	-0.20	15.035333	15.035333	2.013495	2.010521	-0.14
0.099875	0.905498	20.023237	20.007525	-0.07	6.021524	6.016799	-0.04	5.292831	5.287053	-0.10	15.018733	15.018773	2.007175	2.005600	-0.07
0.099984	0.900705	20.002942	20.00963	-0.009	6.002752	6.002158	-0.009	5.257392	5.256668	-0.01	15.002400	15.002400	2.000917	2.000719	-0.009

Table-3: EOD on the propagation of cylindrical hydro magnetic SS in a SGG; Correction percentages for flow variables : Taking p_0/p_1 = 0.9 at r = 0.1, γ=1.4, D = 20.04 w = 1, U/a_0 = 20 at r - 0.1, ξ = 1.5, K_1 = 014387231, K = -3.487226, A = 0.027806, L = 1.77778, $\chi(\xi)$ = 0.010145, B_1 = 0.032064, B_2 = -53.259668, A* = -0.013313, B_1* = -0.467311, B_2* = -0.467311, T = -369. 678121, T = -540.856874

r			U/a„			U/a'			P/P'			P/P^1		u/a^1	
	p_0/p_1,	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	FP	EOD	CP%
0.099546	0.920013	20.084824	20.027365	-0.28	6.078227	6.060838	-0.28	5.400729	5.379217	-0.39	15.068411	15.068411	2.026076	2.020279	-0.28
0.099656	0.915148	20.064205	20.020725	-0.21	6.059257	6.046126	-0.21	5.364484	5.348303	-0.30	15.051778	15.051778	2.019752	2.015375	-0.21
0.099765	0.910338	20.043818	20.014151	-0.14	6.040472	6.031532	-0.14	5.328737	5.317762	-0.20	15.035333	15.035333	2.013491	2.010511	-0.14
0.099875	0.905495	20.023290	20.007523	-0.07	6.021530	6.016789	-0.07	5.292833	5.287034	-0.10	15.018733	15.018773	2.007177	2.005596	-0.07
0.099984	0.900706	20.002994	20.000963	• -0.01	6.002771	6.002162	-0.01	5.257416	5.256674	-0.01	15.002400	15.002400	2.000924	2.000721	-0.01

Table-4: EOD on the propagat ion of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables : Taking rf = 0.9 at y = 0.1,
u

γ = 1.4, D = 20.0 w = 1, U/a_0 = 20 at r = 0.1, ξ = 4.5, K_1 = 0.439600, K = -3.49600, A = 1.130218, L = 17.3333, $\chi\,(\xi)$ = 0.650677,

B_1 = -0.024553, B_2 = -14.028854, A* = 2.186404, B_1* = 0.369705, B_2* = 50.880936.T = -111.007279,T = -1.307673

r		U/a$_0$			U/a'			P/P^1			P/P'		u/a'		
	p_0/p_1	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	FP	EOD	CP%
0.099546	0.920049	20.067793	20.071738	0.01	6.073192	6.074386	0.01	257.03551	257.13030	0.03	45.205232	45.205232"	4.723594	4.724522	0.019
0.099656	0.915174	20.052137	20.054298	0.01	6.055698	6.056351	0.01	255.29954	255.35115	0.02	45.155334	45.155334	4.709987	4.710495	0.010
0.099765	0.910355	20.035582	20.037054	0.007	6.038047	6.038490	0.007	253.56210	253.59704	0.003	45.105999	45.105999	4.696259	4.696603	0.007
0.099875	0.905502	20.018909	20.019688	0.003	6.020236	6.020470	0.003	251.81691	251.83529	0.007	45.056320	45.056320	4.682406	4.682588	0.003
0.099984	0.900703	20.002423	20.002517	0.0004	6.002590	6.002618	0.0004	249.56691	249.59836	0.0004	45.007201	45.007201	4.668681	4.668703	0.0004

Table-5: EOD on the propagation of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables : Taking p_0/p_1= 0.9 at γ = 0.1,

γ = 1.4, D = 20.0 w U/a_0 = 20 at r = 0.1, ξ = 5.9, K_1 = 0.439600, K_1 = -3.49600, A = 1.879625, L = 344, χ £) = 0.13023, B_1 = -0.035723, B_2 = -341.963777,

A* = 3.850291, B_1 = 0.017033, B_2= 89.576424.T = 2711.408814, T = 7504.209249

r		U/a$_0$			U/a'			p/p'			p/p'		u/a'		
	p_0/p_1	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	CP%	FP	EOD	FP	EOD	CP%
0.099546	0.920049	19.993325	20.169789	0.87	6.050655	6.004059	0.87	364.39206	365.25125	0.23	59.269028	59.269028	5.025120	5.069473	0.87
0.099656	0.915174	19.994998	20.128533	0.66	6.038442	6.078770	0.66	362.46926	363.10783	0.17	59.203661	59.203661	5.014977	5.048470	0.66
0.099765	0.910355	19.996621	20.087726	0.45	6.026305	6.053761	0.45	360.56831	361.00122	0.11	59.135977	59 135977	5.004897	5.027700	0.45
0.099875	0.905502	19.998224	20.046620	0.24	6.014015	6.028569	0.24	358.65372	358.88226	0.06	59.073842	59.073842	4.994690	5.006778	0.24
0.099984	0.900703	19.999779	20.005962	0.03	6.001796	6.003652	0.03	356.76022	356.78923	0.008	59.009442	59.009442	4.984542	5.986084	0.03

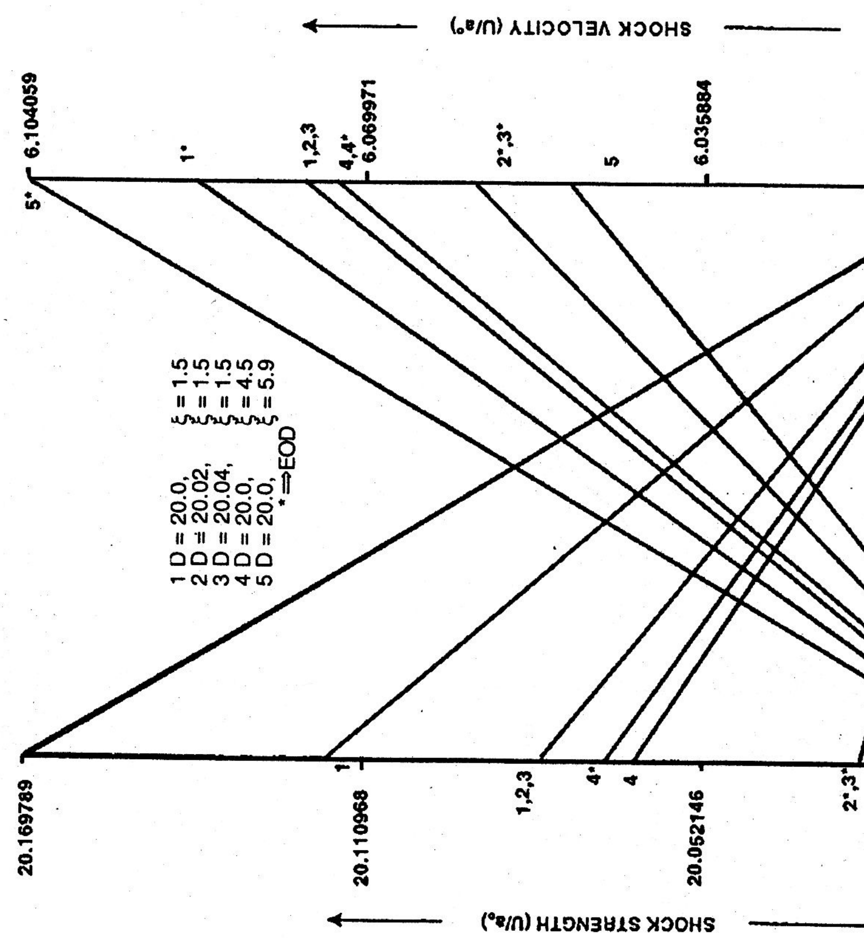

Fig. 1. Variation of schock strength (-) and shock velocity (---) with propagation distance for cylindrical SS in SGG

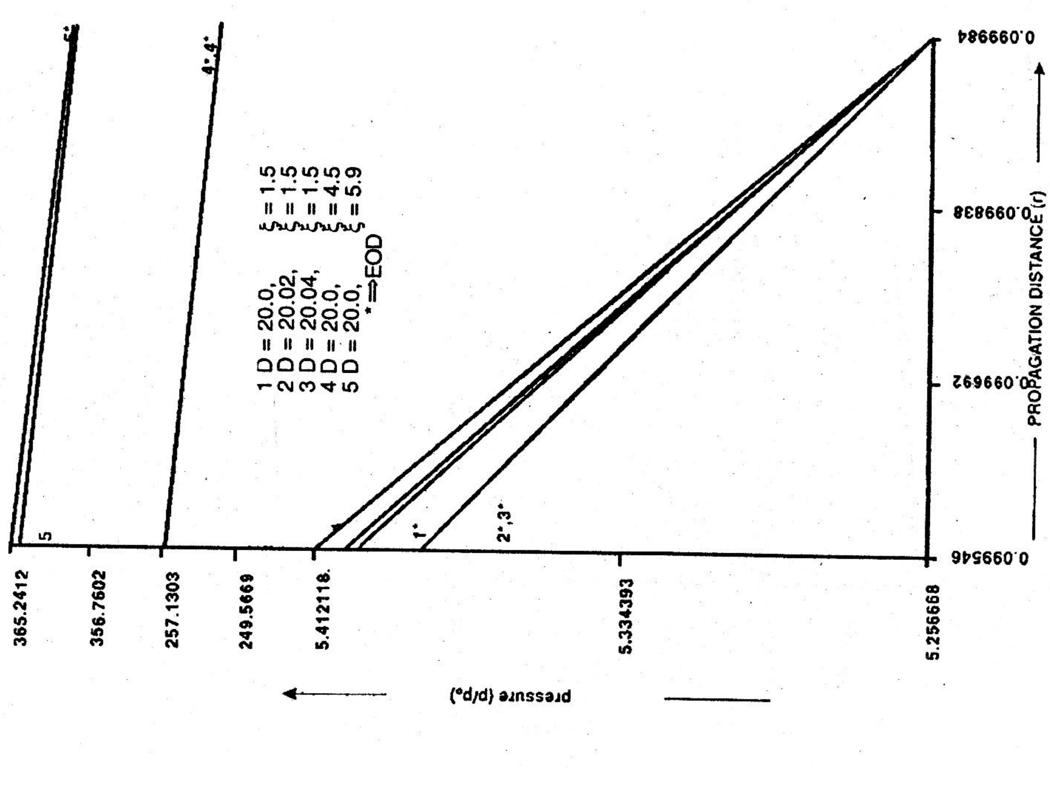

Fig. 2. Variation of pressure immediately behind the shock with propagation distance for cylindrical SS in SGG.

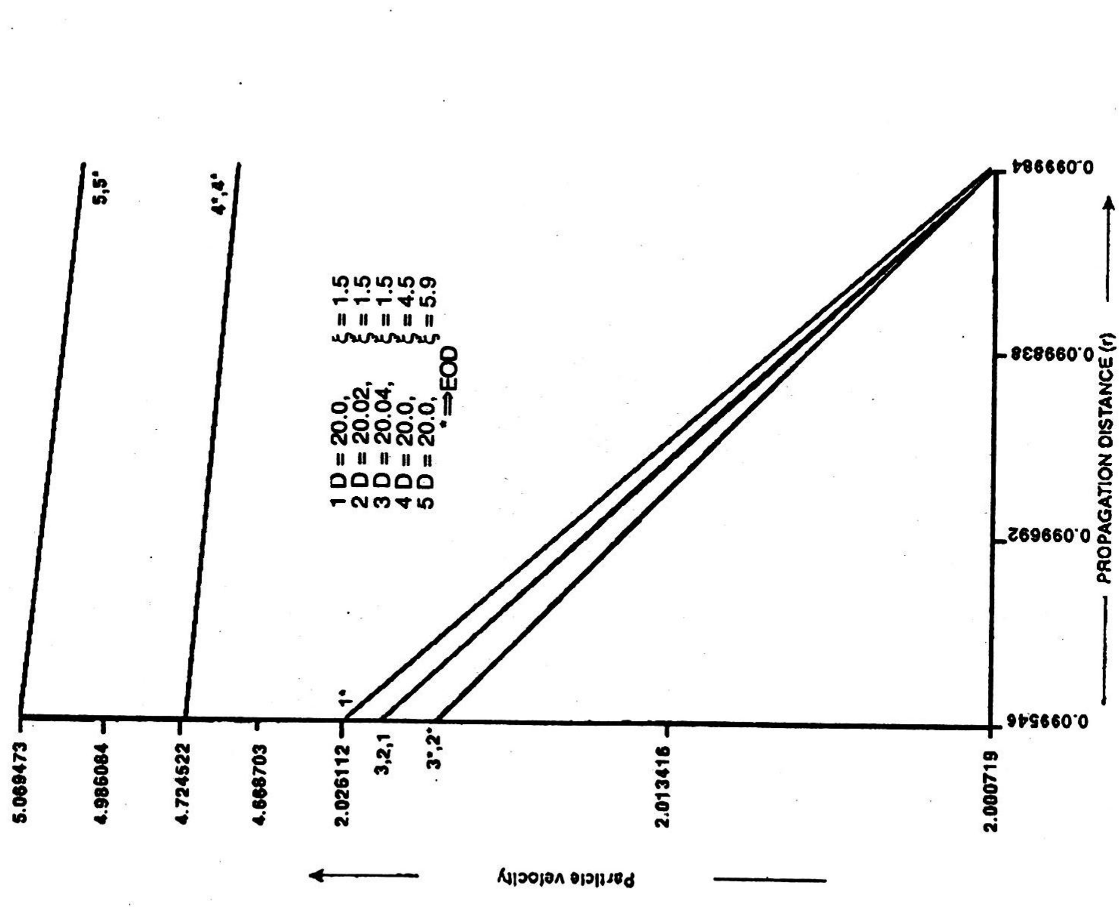

Fig. 3. Variation of particle velocity immediately behind the shock with propagation distance for cylindrical SS in SGG.

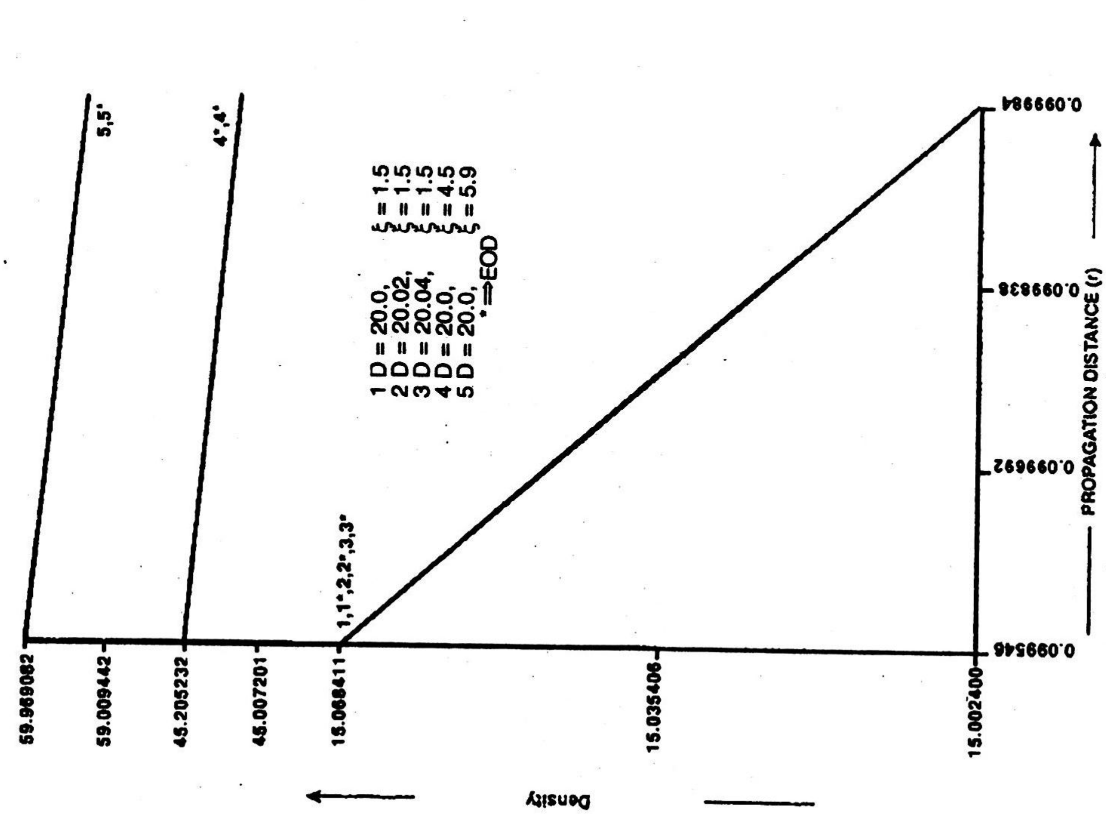

Fig. 4. Variation of density immediately behind the shock with propagation distance for cylindrical SS in SGG

$$B_1^* = \left\{ \frac{\chi(\xi)(\xi-1)}{(\xi-1)+\sqrt{\chi\xi(\xi)}} - \frac{\chi(\xi)}{\gamma} w \right\}$$

$$B_2^* = \left\{ \frac{\xi\sqrt{\chi\xi(\xi)}}{(\xi-1)+\sqrt{\chi\xi(\xi)}} + L \right\}$$

The above equations can be written as

$$\frac{dU^2}{dr} + \frac{B_1^*}{r}U^2 - B_2^* \frac{G2\pi\rho^1}{(2-w)} r^{-w} = 0 \quad (15)$$

Where $B_1^* = \frac{B_1^+}{A^*}$ and $B_2^* = \frac{B_2^*}{A^*}$

For the C_t disturbances generated by the shock, the fluid velocity increment using (15) into (3) may be expressed as

$$du_+ = \frac{\xi-1}{\xi} \cdot \frac{1}{2U}\left\{ -B_1^* U^2 \frac{dr}{r} + \frac{B_2^* 2\pi G\rho^1}{(2-w)} r^{-w} dr \right\} \quad (16)$$

Now in presence of both C. and C_+ distruahces the fluid velocity increment behind the shock will be related as

$$du_- + du_+ = \frac{\xi-1}{\xi} du \quad (17)$$

Substituting (13), (16) into (17), we get

$$\frac{dU^2}{dr} + (B_1 + B_1^*)\frac{U^2}{r} + \frac{(B_2+B_2^*)}{(2-w)} G2\pi\rho^1 r^{-w} \} = 0 \quad (18)$$

on integration (18), we get

$$U^2 = \left[T^1 r^{-(B_1+B_1^*)} - \frac{2\pi(B_2+B_2^*)}{D(2-w)} \frac{r^{1-w}}{(1+B_1-w)} \right] \quad (19)$$

Where T^1 is a constant of integration $T^1 = T.a^{12}$

It describes the propagation parameter U^2 which includes the eod behind the flow on the motion of the shock.

Analytical Expression for flow Variables :

Substituting the equation (12) and (19) into the shock condition (3) we get, respectively, for both FP and flow variables having included the FP

$$\frac{U}{a_0} = \left[\left\{ Tr^{-B_1-w} - \frac{2\pi B_2}{D(2-w)} \frac{r^{1-w}}{(1-w+B_1)} \right\} \frac{1}{K}\left(1 - \frac{K_1}{K} r^{1-2w}\right) \right]^{1/2} \quad (20)$$

$$\frac{u}{a_0} = \left\{ Tr^{-B_1} - \frac{2\pi B_2}{D(2-w)(1-w+B_1)} \right\}^{1/2} \quad (21)$$

$$\frac{p}{p^1} = \left[\chi(\xi)\left\{ Tr^{-B_1-w} - \frac{2\pi B_2}{D(2-w)} \frac{r^{1-2w}}{(1-w+B_1)} \right\} + L(K + K_1 + r^{1-w}) \right]$$

$$\frac{\rho}{\rho^1} = r^{-w}\xi \quad (23)$$

$$\frac{u}{a^1} = \frac{\xi-1}{\xi}\left[\left\{ Tr^{-B_1} - \frac{2\pi B_2}{D(2-w)} \frac{r^{1-w}}{(1-w+B_1)} \right\} \right]^{1/2}$$

EOD

$$\frac{U}{a_0} = \left[\left\{ Tr^{-(B_1+B_1^*)-w} - \frac{2\pi(B_2+B_2^*)}{D(2-w)} \frac{r^{1-w}}{(1-w+B_1+B_1^*)} \right\} \frac{1}{K}\left(1 - \frac{K_1}{K} r^{1-2w}\right) \right]^{1/2}$$

$$\frac{u}{a_0} = \left\{ Tr^{-(B_1+B_1^*)} - \frac{2\pi(B_2+B_2^*)}{D(2-w)(1-w+B_1+B_1^*)} \right\}^{1/2} \quad (26)$$

$$\frac{p}{p^1} = \left[\chi(\xi)\left\{ Tr^{-(B_1+B_1^*)-w} - \frac{2\pi(B_2+B_2^*)}{D(2-w)} \frac{r^{1-2w}}{(1-w+B_1+B_1^*)} \right\} + L(K + K_1 + r^{1-w}) \right]$$

$$\frac{\rho}{\rho^1} = r^{-w}\xi \quad (28)$$

$$\frac{u}{a^1} = \frac{\xi-1}{\xi}\left[\left\{ Tr^{-(B_1+B_1^*)+w} - \frac{2\pi(B_2+B_2^*)}{D(2-w)} \frac{r^{1-w}}{(1-w+B_1+B_1^*)} \right\} \right]^{1/2} \quad (29)$$

RESULTS AND DISCUSSION

Numerical estimates of flow variables for both FP and having included the eod, have been computed only at those location of the shock front which are persmitted by the initial entropy condition in the unperturbed state are shown in Table 1,2,3,4 and 5 and Fig, 1.2,3 and 4. Taking: (i) U/a_0 = 20 at r=0.1 for γ=1.4, D= 20.00 w=1.0,ξ = 1.5, (ii) U/a_0 = 20 at r=0.1 for γ=1.4, D= 20.02 w=1.0,ξ = 1.5, (iii) U/a_0 = 20 at r=0.1 for γ =1.4 , D= 20.04 w=1.***0,*** ξ = 1.5, (iv) U/a_0 = 20 at r=0.1 for γ =1.4, D= 20.0 w=1.0, ξ = 4.5, (v) U/a_0 = 20at r=0.1 for γ =1.4, D= 20.0 w=1.0, ξ = 5.9

REFERENCES

1. Chester W, *Philos Marg* **45**, 293 (1954)
2. Chisnell R F, *Proc Roy Sec (London)* **A-232,** 350 (1955)
3. Whitham *G B , J Fluid Mech,* **4,337** (1958)
4. Kumar S and Chaturani P II, *Nuovo Cimento,* **356**, 297 (1980)
5. Kumar S and Prakash R, *Proc Nat Soc Fluid Mech Fluid Power,* **615**, (1) 277 (1986)
6. Chaturani PII. *Nuovo Cimento,* **3**,56,297 (1980)
7. W D Hayes, *J Fluid Mech,* **32**,305, (1968)
8. Yousaf M, *Phys Fluid,* **25**(1), 45 (1982)