

Fourier transform in speckle photography for measuring the refractive index for a transparent plate

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ABSTRACT

This paper reports on the application of Fourier transform to speckle photography. The simple technique here enables the measurement of the refractive index of a transparent plate. A simple mathematical approach has been applied utilizing the resulting data to estimate the refractive index of the plate.

Key words: Speckle photography, Fourier transform, refractive index estimation.

INTRODUCTION

The principal problem in fringe-pattern analysis is how to reveal phase information from a fringe pattern by separating it from background intensity and local contrast variation. One solution to this problem, is the Fourier transform method (FTM) first proposed by Takeda, Ina and Kabayashy.¹ It has been a wide-spread technique in fringe pattern analysis,² and has been incorporated in many interferometric methods with subfringe sensitivity, such as moiré profilometry and contouring,³ holographic interferometry,⁴ grating interferometry,⁵ wave length shifting interferometry,⁶ and recently speckle photography.⁷ Speckle photography has been proved to be a promising tool for in-plane and out-of-plane displacement, strain, and slope measurements, as well as contouring⁹⁻¹¹.

The aim of this paper is to apply the Fourier transform to speckle photography for introducing a simple technique, capable of measuring the refractive index of a transparent plate. New formulae is obtained and their validity is substantiated by experiment.

Theoretical Analysis

Consider first the transmission of a plane wave through a transparent plate of refractive index n and thickness d surrounded by free space (Fig.1).

Let $U(x,y,d)$, and $U(x,y,0)$ are the complex amplitudes of the plane wave after and before transmission respectively. Therefore the ratio $t(x,y) = U(x,y,d) / U(x,y,0)$ represent the complex amplitude transmittance of the plate.

$$U(x, y, d) = ae^{j\phi} e^{-jkd} \quad \dots(1)$$

Where α and ϕ are the amplitude and phase of the incident wave respectively.

The transmitted wave illuminates a scattering surface lies in the (ζ, η) plane. Any point on the observation plane (X, Y) is placed at a distance Z in front of this surface will receive contributions from all points on the surface.

Assume that P_s is any point on the scattering surface plane, P_o is any point on the

observation plane and r_{os} is the distance between P_s and P_o making an angle θ with Z .

According to Fresnel principle of diffraction, we can write wavefield across the (X, Y) plane as follows

$$U(p_o) = \frac{1}{j\lambda} \iint a e^{j\phi} e^{-jkd} U(p_s) \frac{e^{jkr_{os}}}{r_{os}} \cos \theta ds \quad \dots(2)$$

Where

$$\cos \theta = \frac{Z}{r_{os}}$$

This equation can be written as:

$$U(X, Y) = \frac{Z}{j\lambda} a e^{j\phi} \iint U(\xi, \eta) e^{-jkd} \frac{e^{jkr_{os}}}{r_{os}} d\xi d\eta \quad \dots(3)$$

Where

$$r_{os} = \sqrt{Z^2 + (X - \xi)^2 + (Y - \eta)^2} \quad \dots(4)$$

Let m be a number less than unity and consider the equation

$$\sqrt{1+m} = 1 + \frac{1}{2}m - \frac{1}{8}m^2 + \dots \quad \dots(5)$$

Then

$$r_{os} = Z \sqrt{1 + \left(\frac{X - \xi}{Z}\right)^2 + \left(\frac{Y - \eta}{Z}\right)^2} \quad \dots(6)$$

$$r_{os} \approx 1 + \frac{1}{2} \left(\frac{X - \xi}{Z}\right)^2 + \frac{1}{2} \left(\frac{Y - \eta}{Z}\right)^2 \quad \dots(7)$$

$$U(X, Y) = \frac{Z}{j\lambda} a e^{j\phi} \iint U(\xi, \eta) e^{-jkd} \frac{e^{jkr_{os}}}{r_{os}} \frac{e^{j\frac{k}{2Z}((X-\xi)^2 + (Y-\eta)^2)}}{1 + \frac{((X-\xi)^2 + (Y-\eta)^2)}{2Z^2}} d\xi d\eta \quad \dots(8)$$

$$U(X, Y) = \frac{1}{j\lambda Z} a e^{j\phi} e^{-jkd} \iint U(\xi, \eta) \frac{e^{j\frac{k}{2Z}((X-\xi)^2 + (Y-\eta)^2)}}{1 + \frac{((X-\xi)^2 + (Y-\eta)^2)}{2Z^2}} d\xi d\eta \quad \dots(9)$$

The terms $((X - \xi)/Z)^2$ and $((Y - \eta)/Z)^2$ are very small and their effect on the wave amplitude can be neglected, then

$$U(X, Y) = \frac{1}{j\lambda Z} a e^{j\phi} e^{-jkd} \iint U(\xi, \eta) e^{j\frac{k}{2Z}((X-\xi)^2 + (Y-\eta)^2)} e^{-j\frac{k}{2Z}((X-\xi)^2 + (Y-\eta)^2)} d\xi d\eta \quad \dots(10)$$

$$\text{If } Z \gg (\xi^2 + \eta^2)/2$$

then

$$U(X, Y) = \frac{1}{j\lambda Z} a e^{j\phi} e^{-jkd} e^{j\frac{k}{2Z}((X^2 + Y^2))} \iint U(\xi, \eta) e^{-j\frac{k}{2Z}((X-\xi)^2 + (Y-\eta)^2)} d\xi d\eta \quad \dots(11)$$

Refractive index measurement

To measure the refractive index of the transparent plate, the plate is put on a graduated rotatable disc. Spatially coherent light transmitted through the plate and illuminated a rough surface. The obtained speckle is recorded twice, once before the rotation of the transparent plate and once after the rotation. Then the two images are combined digitally "added" and the resultant image will contain a pair of identical speckle pattern separated by a distance $\Delta\xi$.

The displacement is displaced in the form of fringe pattern by applying FFT to the resultant image. So a bright spot surrounded by a speckle pattern modulated by a sinusoidal fringes can be observed. The bright central spot is formed by the undiffracted light. While the sinusoidal fringes are formed because each pair of corresponding speckles acts as a pair of identical sources of coherent light which form Young's fringes. These fringes are separated by a distance depending on the value of the rotation of the transparent plate.

Using equation (11), and put and

$$U(X, Y) = \frac{1}{j\lambda Z} a e^{j\phi} e^{-jkd} e^{j\frac{k}{2Z}((X^2 + Y^2))} \iint U(\xi, \eta) e^{-j\frac{k}{2Z}((X-\xi)^2 + (Y-\eta)^2)} d\xi d\eta \quad \dots(13)$$

The field before the rotation of the transparent plate can be represented by:

$$U_1(X, Y) = \frac{1}{j\lambda Z} a e^{j\phi} e^{-jkd} e^{j\frac{k}{2Z}((X^2 + Y^2))} F\{U(\xi, \eta)\} \quad \dots(14)$$

Where $F\{U(\xi, \eta)\}$ is the Fourier transform function. If the transparent plate rotated with an angle β , the incident waves refracted with an angle γ .

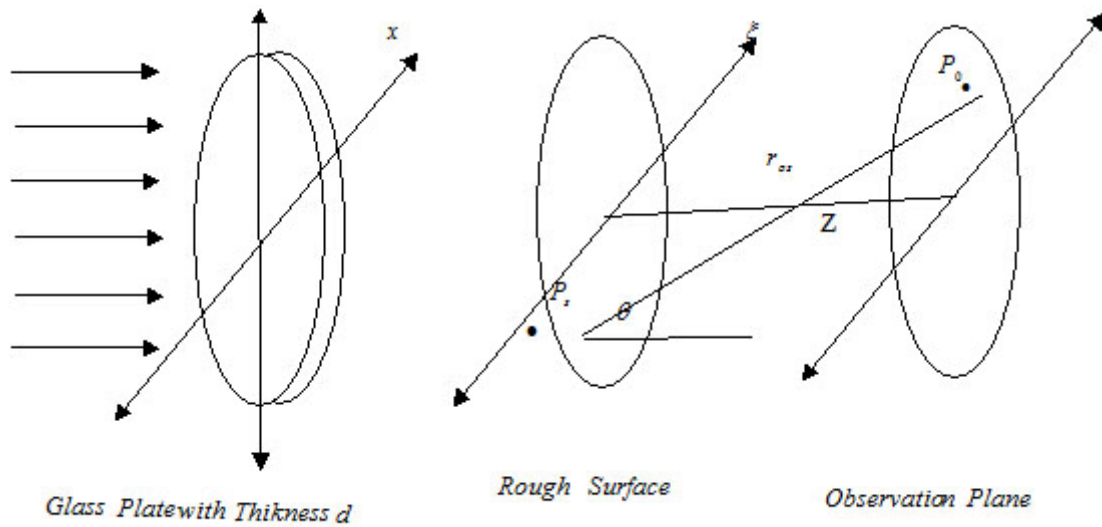


Fig. 1: The construction of the speckle pattern

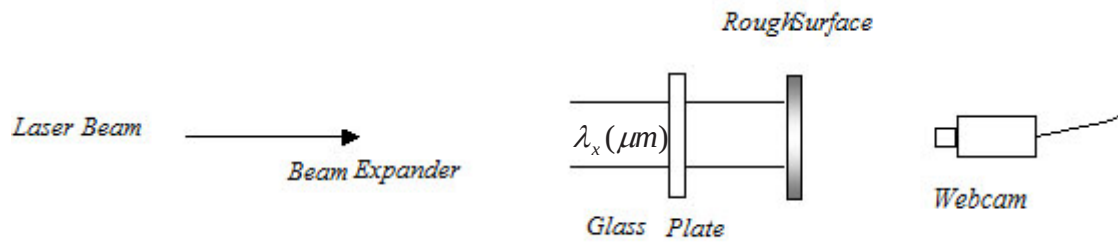
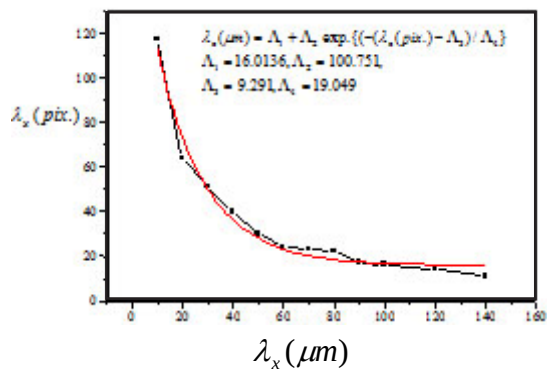
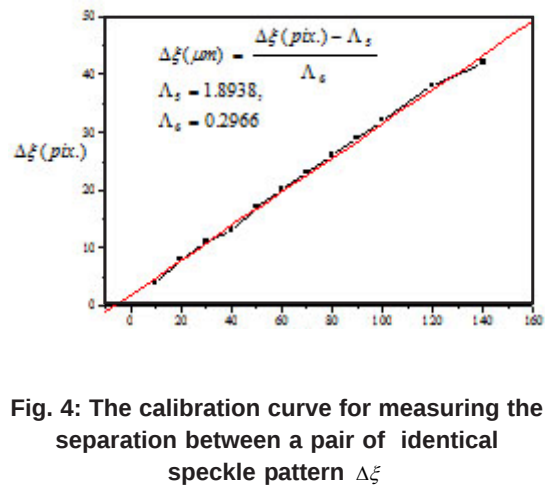


Fig. 2: Interferometric technique for measuring refractive index of a transparent plate.

Fig. 3: The calibration curve for measuring the fringe spacing; λ_x in the x-axis directionFig. 4: The calibration curve for measuring the separation between a pair of identical speckle pattern Δ_x^s

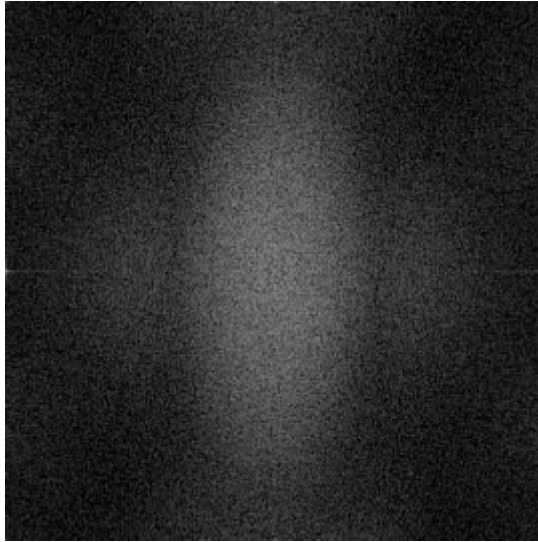


Fig. 5: Image of fringes in the spectral field: the transparent plate was rotated by an angle 0.5°

The complex amplitude inside the plate is now proportional to $\exp(-jkd / \cos \gamma)$. So that the complex amplitude transmittance of the plate:

$$t(x, y) = \exp(-jkd / \cos \gamma) \quad \dots(14)$$

The field after rotation can be represented by:

$$U_1(X, Y) = \frac{1}{j\lambda Z} e^{j\frac{\pi}{2}} e^{-\frac{j\pi}{2} \frac{X^2 + Y^2}{Z}} e^{j\frac{\pi}{2} \frac{X^2 + Y^2}{Z}} \iint U(\xi - \Delta\xi, \eta) e^{-j2\pi(\xi - \Delta\xi)(X - \Delta X) + j2\pi(\eta - \Delta\eta)(Y - \Delta Y)} d\xi d\eta \quad \dots(15)$$

$$U_2(X, Y) = \frac{1}{j\lambda Z} e^{j\frac{\pi}{2}} e^{-\frac{j\pi}{2} \frac{X^2 + Y^2}{Z}} e^{j\frac{\pi}{2} \frac{X^2 + Y^2}{Z}} \iint U(\xi - \Delta\xi, \eta) e^{-j2\pi(\xi - \Delta\xi)(X - \Delta X) + j2\pi(\eta - \Delta\eta)(Y - \Delta Y)} d\xi d\eta \quad \dots(16)$$

Assume that the displacement is in one direction (ξ -axis). Hence;

$$U_2(X, Y) = \frac{1}{j\lambda Z} e^{j\frac{\pi}{2}} e^{-\frac{j\pi}{2} \frac{X^2 + Y^2}{Z}} e^{j\frac{\pi}{2} \frac{X^2 + Y^2}{Z}} e^{j2\pi \Delta\xi (X - \Delta X)} F\{U(\xi, \eta)\} \quad \dots(17)$$

By adding the two Equations (13) and (17)

$$U_1(X, Y) + U_2(X, Y) = \frac{1}{j\lambda Z} e^{j\frac{\pi}{2}} e^{-\frac{j\pi}{2} \frac{X^2 + Y^2}{Z}} F\{U(\xi, \eta)\} (e^{-j2\pi \Delta\xi (X - \Delta X)} + e^{j2\pi \Delta\xi (X - \Delta X)}) \quad \dots(18)$$

The intensity can be written as

$$I = \left(\frac{1}{\lambda Z}\right)^2 F^2\{U(\xi, \eta)\} \{e^{-j2\pi \Delta\xi (X - \Delta X)} + e^{j2\pi \Delta\xi (X - \Delta X)}\}^2 \quad \dots(19)$$

$$I = \left(\frac{1}{\lambda Z}\right)^2 F^2\{U(\xi, \eta)\} \{2 + 2\cos(kd - \frac{kd}{\cos \gamma} + 2\pi \Delta\xi \omega_x)\} \quad \dots(20)$$

At maximum intensity

$$kd - \frac{kd}{\cos \gamma} + 2\pi \Delta\xi \omega_x = 2m\pi \quad \dots(21)$$

Let $m=1$, we get:

$$\Delta\xi \omega_x = 1 + \frac{d}{\lambda} \left(\frac{1}{\cos \gamma} - 1\right) \quad \dots(22)$$

Put

$$\Delta\xi \frac{2\pi}{\lambda_x} = 1 + \frac{d}{\lambda} \left(\frac{1}{\cos \gamma} - 1\right) \quad \dots(23)$$

Where λ is the wavelength of the used light and λ_x represent the fringe spacing in the x-direction.

From the Snell's law, $n' \sin \beta = n \sin \gamma$, where $n'=1$ is the refractive index of the air, and n is the refractive index of the glass plate. By substitution from the Snell's law into Equation (23). The refractive index can be given by:

$$n = \frac{\sin \beta}{\sin \left[\cos^{-1} \left(\frac{1}{\left(\frac{2\pi}{\lambda_x} \Delta\xi - 1 \right) \frac{d}{\lambda} + 1} \right)} \right]} \quad \dots(24)$$

RESULTS

The schematic diagram of the experimental set-up is shown in Fig.(2). Spatially coherent light from an unpolarized He-Ne laser source is employed after being expanded and collimated for illuminating a transparent rough

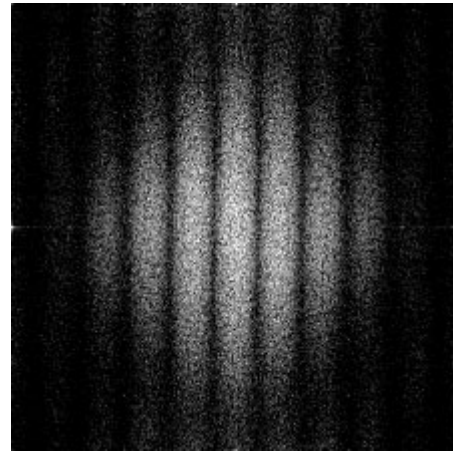
surface. The rough surface have rms 9.6 μm . To study the refractive index of a transparent plate (glass) of thickness 5.8 mm, the glass plate putted on a graduated rotatable disc in front of the collimated light. Having a vernier, the smallest graduation can be read to 0.1 degree. The sensor of the webcam camera has received directly the two speckle pattern before and after the rotation of the glass plate. The unit to be measured on monitor is pixel. So we must calibrate the magnification before the test. Fig. 3 shows the calibration curve for measuring the value of λ_x in μm . Fig.4 shows the calibration curve for measuring the value of λ_y . The values $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5$ and Λ_6 which are shown in the equations of the calibrations of λ_x and λ_y are considered as constants depending on the calibration process.

The measuring steps for getting the refractive index are as follows:

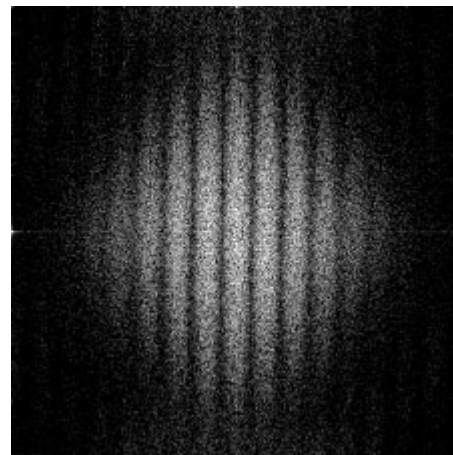
1. Adjusting the webcam until we see a clear speckle image on the monitor, and recording the initial speckle.
2. Making the transparent glass plate rotates 0.5 degree and recording the speckle pattern again. From the recorded two images and by using the software program (Image J Program), we can obtain the value of λ_x .
3. Combining the two images into one and computing their two-dimensional Fourier transform, we get the fringe image in the spectral field (Fig. 5) for angle of rotation 0.5 degree.
4. Scanning Fig. 5 along the λ_x -axis ($\lambda_y=0$), we can get the distribution of the light intensity, so the value of λ_x can be measured.
5. After measuring the value of λ_x and λ_y , we substitute in Eq. 24, we can get the refractive index of the glass plate.

Figure 6(a, b, c) show the fringe images in the spectral field for different angles of rotation 2°, 3°, and 4° respectively.

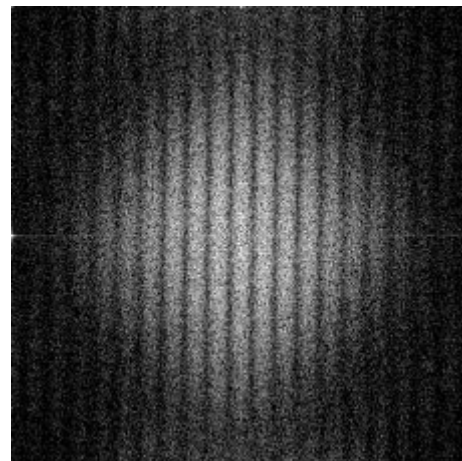
The experimental results obtained for measuring the refractive index with different angles of rotation are shown in Table (1). From the table we can see, the good accuracy of measurement even for different angles.



(a)



(b)



(c)

Fig. 6: (a,b,c) Images of fringes in the spectral field: the transparent plate was rotated by an angle 2°, 3°, and 4° respectively

Table 1. The experimental results obtained for measuring the refractive index with different angles of rotation of the transparent plate

Angle of rotation (β)	Angle of refraction (γ)	Refractive index (n)	Percentage error
0.5°	0.329°	1.516	2.19
2.5°	1.1.649°	1.515	2.25
3.5°	2.228°	1.570	-1.29
4°	2.622°	1.524	1.67

CONCLUSION

A simple technique has been demonstrated for determining the refractive index of a transparent plate by measuring the angle of

refraction of monochromatic light transmitted through the plate. Furthermore, a new formulae was presented by which the refractive index is obtained with high accuracy of high accuracy of measurement.

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