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Solitary wave solutions for a generalized Broer-Kaup system

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ABSTRACT

In this work we take into consideration a (2+1)-dimensional higher order Broer-Kaup system. Amodification of homogenous balance method (HB) is suggested to search for periodic solution and solitary wave solution of Broer-kaup system of nonlinear equations.

Key words: Kadomtsev-Petviashvili (KP) equation, the (2+1)-dimensional higher order Broer-Kaup system, homogenous balance method, nonlinear partial differential equations, exact solution.

INTRODUCTION

In the past few decades, there has been noticeable progress in the construction of exact solutions for non-linear partial differential equations which has long been a major concern for both mathematicians and physicists. This progress in finding exact solutions to non-linear differential equations, when they exist, is very important to the understanding of most non-linear physical phenomena. For instances, the non-linear wave phenomena observed in fluid dynamics, plasma and optical fibers are often modeled by the bell-shaped sech solutions and the kink-shaped tanh solutions. There is many powerful method for finding solution of non linear partial differential equations such as inverse scattering transform[1], Hirota;s bilinear methods[2], Painleve expansion[3], Homogeneous Balance method[4], Backlund transformation method[5], tansh method[6], Miura transformation method[7], Jacobi elliptic function expansion method[8], F-expansion method[9] and so on.

In this paper, we consider the following (2+1)-dimensional higher order Broer-Kaup system

$$u_t + 4(u_{xx} + u^3 - 3 \quad uu_x + 3uw + 3p)_x = 0$$
$$v_t + 4(v_{xx} + 3vu^2 + uv_x + 3vw)_x = 0$$
$$w_y - v_x = 0$$
$$p_y - (uv)_x = 0$$

which is obtained from the Kadomtsev-Petviashvili (KP) equation by the symmetry constraint[10]. In solving system (1), given by Li *et al*[11] via a transformation and tanh-function method to obtain many new exact solutions, Jain *et al* [12] reduced a system to a simple (1+1)-dimensional non-linear evolution equation through a simple transformation. By using the new generally projective Riccati equation expansion method, they explore many families of soliton-like and periodic solutions for it. Recently, Li *et al* [13] have obtained some new types of multisoliton solutions for system (1) by using some simple transformations as $v = u_y,\ w = u_x + \alpha(x,\ t),\ p = w_x + \beta$ and homogenous balance method.

The following is a summary of our paper: in section 2 we discuss the solitary wave solution of system (1), this technique is based to get many cases of travelling-wave solutions.

Travelling wave solutions

Our method is summed up as follows[14-17]: For a given nonlinear partial differential equations with some physical fields $u_i(x,\ y,\ t)$ $(i = 0,\ 1,\ 2, \ldots)$ in three variables x, y, t

$$F_i(u_i,\ u_{it},\ u_{iy},\ u_{ix},\ u_{itt},\ u_{ixx}, \ldots) = 0 \qquad ..(2)$$

we consider it's travelling wave solutions in the following form

$$u_i(x,\ y,\ t) = u_i(\xi),\quad \xi = \lambda_1 t + \lambda_2 x + \lambda_3 y$$

where λ_1, λ_2 and λ_3 are arbitrary constants to be determined later. Equations (2) are reduced to nonlinear ordinary differential equations

$$G_i(u_i,\ u_i',\ u_i'', \ldots) = 0 \qquad ...(4)$$

where " / " n denote $\frac{d}{d\xi}$. We now seek the solution of (3) in the form

$$u_i(x,\ y,\ t) = a_0 + \sum_{i=1}^{m} a_i g^i(\xi) + \sum_{j=1}^{m} b_j g^{j-1} f(\xi) \qquad ...(5)$$

where $a_0,\ a_i,\ b_j, (i,\ j = 1,\ 2, \ldots m)$ are constant to be determined later. The parameter m can be determined by balancing the highest order derivative terms with the nonlinear terms in equation (3) and $f(\xi)$, $f(\xi)$ satisfy the following extended Riccati equations

$$\begin{aligned} f'(\xi) &= -g(\xi) f(\xi) \\ g'(\xi) &= 1 - g^2(\xi) - r f(\xi) \\ g^2(\xi) &= 1 - 2 r f(\xi) + (r^2 + \delta) f^2(\xi) \end{aligned} \qquad ....(5)$$

$\delta = \pm 1$ and r arbitrary constant. Equation (5) admits the following solutions

when δ=1

$$f(\xi) = \frac{1}{\sinh(\xi) + r},\quad g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi) + r} \qquad ...(6)$$

when δ = 1

$$\begin{aligned} f(\xi) &= \frac{4}{5\cosh(\xi) + 3\sinh(\xi) + r}, \\ g(\xi) &= \frac{5\sinh(\xi) + 3\cosh(\xi)}{5\cosh(\xi) + 3\sinh(\xi) + r} \end{aligned} \qquad ...(7)$$

or

$$f(\xi) = \frac{1}{\cosh(\xi) + r},\quad g(\xi) = \frac{\sinh(\xi)}{\cosh(\xi) + r} \qquad ...(8)$$

Then substituting (4) along with (5) in equation (3) yields a set of equations for $f^i g^j$ $(i = 0,\ 1,\ j = 0,\ 1,\ 2, \ldots)$. Setting the coefficient of $f^i g^j$ to zero we get a set of over-determined system of nonlinear algebraic equations and by using Maple program we can get the values of $\lambda_i,\ a_0, a_i,\ b_i$ $(i,\ j = 1, \ldots,\ m)$ and hence we can obtain many exact solutions of (3) according to (6)-(8).

Now assuming the transformation where $u(x,\ y,\ t) = U(\xi),\ v(x,\ y,\ t) = V(\xi),\ w(x,\ y,\ t) = W(\xi),$ $p(x,\ y,\ t) = P(\xi)$ where $\xi = \lambda_1 t + \lambda_2 x + \lambda_3 y$ then system (1) reduced to

$$\begin{aligned} \lambda_1 U' + 12\lambda_2 P' + 12\lambda_2 U^2 U' + 12\lambda_2 W U' - 12\lambda_2^2 U' + 12\lambda_2 U W' - 12\lambda_2^2 U U'' + 4\lambda_2^3 U''' &= 0 \\ \lambda_1 V' + 24\lambda_2 U V U' + 12\lambda_2 U^2 V' + 12\lambda_2 W V' + 4\lambda_2^2 U' V' + 12\lambda_2 V W' + 4\lambda_2^2 U V'' + 4\lambda_2^3 V''' &= 0 \\ -\lambda_3 V' + \lambda_2 W' &= 0 \\ \lambda_2 P' - \lambda_1 V U' - \lambda_1 U V' &= 0 \end{aligned}$$

...(9)

Now according to (4)

$$u(x,\ y,\ t) = U(\xi) = a_0 + \sum_{i=1}^{m} a_i g^i(\xi) + \sum_{j=1}^{m} b_j g^{j-1} f(\xi))$$

$$v(x,\ y,\ t) = V(\xi) = c_0 + \sum_{i=1}^{n} c_i g^i(\xi) + \sum_{j=1}^{n} d_j g^{j-1} f(\xi))$$

$$w(x,\ y,\ t) = W(\xi) = e_0 + \sum_{i=1}^{l} e_i g^i(\xi) + \sum_{j=1}^{l} h_j g^{j-1} f(\xi))$$

$$p(x,\ y,\ t) = P(\xi) = k_0 + \sum_{i=1}^{r} k_i g^i(\xi) + \sum_{j=1}^{r} s_j g^{j-1} f(\xi))$$

Balancing the highest order derivative terms with the nonlinear terms of system (9) we get $m = 1,\ n = l = r = 2$

So we can consider

$$\begin{aligned} U &= a_0 + a_1 g + a_2 f \\ V &= b_0 + b_1 g + b_2 g^2 + b_3 f + b_4 fg \\ W &= c_0 + c_1 g + c_2 g^2 + c_3 f + c_4 fg \\ P &= d_0 + d_1 g + d_2 g^2 + d_3 f + d_4 fg \end{aligned} \qquad \text{...(10)}$$

Substituting (10) in (9) and by using (5) we can eliminate all the coefficients of the powe of $f^i g^j$ $(i = 0,\ 1,\ j = 0,\ 1,\ 2,\ldots)$ to get a set of algabraic polynomials of constants and by using Maple program to find the values of a_i, b_j, c_j and d_j where $i = 0,\ 1,\ 2,\ j = 0,\ 1,\ 2,\ 3,\ 4$ [18], then we have the following cases

1. when $\delta=1$ we have

Case 1

$$u_1 = \frac{1}{9}\lambda_2 (7\sqrt{3Z^2+2-Z} - 26) / \sqrt{3Z^2+5+8\sqrt{3Z^2+2-Z}}$$
$$(-2+\sqrt{3Z^2+2-Z}))$$
$$+\sqrt{3Z^2+2-Z}\,\lambda_2 g(\xi)$$

$$v_1 = \frac{1}{18}\lambda_2\lambda_3 (29\sqrt{3Z^2+2-Z} - 73) / (\sqrt{3Z^2+2-Z} + 4)$$
$$+\frac{1}{3}\sqrt{3Z^2+5+8\sqrt{3Z^2+2-Z}}\,\lambda_3\lambda_2 g(\xi)$$

$$w_1 = -\frac{1}{36}(3461656\lambda_2^3\sqrt{3Z^2+2-Z}$$
$$-66993\sqrt{3Z^2+2-Z}\,\lambda_1$$
$$-1437968\lambda_2^3 - 136914\lambda_1) / (\lambda_2(22331\sqrt{3Z^2+2-Z} + 45638))$$
$$+\frac{1}{27}\lambda_2^2 (463\sqrt{3Z^2+2-Z} + 502) g(\xi) / (\sqrt{3Z^2+5+8\sqrt{3Z^2+2-Z}}$$
$$(7\sqrt{3Z^2+2-Z} - 26))$$

$$p_1 = d_0 + \frac{1}{27}\lambda_2^3 (401\sqrt{3Z^2+2-Z} + 38)$$
$$g(\xi) / (7\sqrt{3Z^2+2-Z} - 26)$$
$$+\frac{1}{3}\lambda_2^3\sqrt{3Z^2+5+8\sqrt{3Z^2+2-Z}}$$
$$(\sqrt{3Z^2+2-Z})\, g(\xi)^2$$

$$f(\xi) = \frac{1}{\sinh(\xi)}$$

$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}$$

where $d_0,\ \lambda_1,\ \lambda_2,\ \lambda_3$ are arbitrary constants.

Case 2

$$u_2 = \sqrt{10Z^2+5-7Z}\,\lambda_2 g(\xi)$$

$$v_2 = \frac{11}{225}\lambda_2\lambda_3$$
$$+\frac{11}{15}\lambda_3\sqrt{10Z^2+5-7Z}\,\lambda_2\sqrt{Z^2+1}\, f(\xi)$$

$$w_2 = -\frac{1}{900}(210\sqrt{10Z^2+5-7Z}\,\lambda_2^3 - 106\lambda_2^3 - 75\lambda_1) / \lambda_2$$
$$+\frac{11}{15}\lambda_2^2\sqrt{10Z^2+5-7Z}\sqrt{Z^2+1}\, f(\xi)$$

$$p_2 = d_0 + \frac{11}{225}\sqrt{10Z^2+5-7Z}\,\lambda_2^3 g(\xi) + \frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})\sqrt{Z^2+1}f(\xi)g(\xi)$$

where

$$f(\xi) = \frac{1}{\sinh(\xi)+\sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)+\sqrt{Z^2+1}}$$

and $d_0,\ \lambda_1,\ \lambda_2,\ \lambda_3$ are arbitrary constants.

Case 3

$$u_3 = \sqrt{10Z^2+5-7Z}\,\lambda_2 g(\xi)$$

$$v_3 = \frac{11}{225}\lambda_2\lambda_3 + \frac{11}{15}\sqrt{10Z^2+5-7Z}\,\lambda_2\lambda_3\sqrt{Z^2+1}f(\xi)$$

$$w_3 = -\frac{1}{900}(210\sqrt{10Z^2+5-7Z}\,\lambda_2^3 - 106\lambda_2^3 - 75\lambda_1 + 900\lambda_2 c_2)/\lambda_2 + c_2 g(\xi)^2 + \frac{1}{15}(11\lambda_2^2\sqrt{10Z^2+5-7Z}+30c_2)\sqrt{Z^2+1}f(\xi)$$

$$p_3 = d_0 + \frac{11}{225}\sqrt{10Z^2+5-7Z}\,\lambda_2^3 g(\xi) + \frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})\sqrt{Z^2+1}f(\xi)g(\xi)$$

where

$$f(\xi) = \frac{1}{\sinh(\xi)+\sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)+\sqrt{Z^2+1}}$$

and $d_0,\ c_2,\ \lambda_1,\ \lambda_2,\ \lambda_3$ are arbitrary constants.

Case 4

$$u_4 = (\sqrt{10Z^2+5-7Z})\lambda_2 g(\xi)$$

$$v_4 = \frac{11}{225}\lambda_2\lambda_3 + \frac{11}{15}(\sqrt{10Z^2+5-7Z})\lambda_2\lambda_3\sqrt{Z^2+1}f(\xi)$$

$$w_4 = \frac{1}{900}(763(\sqrt{10Z^2+5-7Z})\lambda_2^3 + 525(\sqrt{10Z^2+5-7Z})\lambda_1 + 205\lambda_2^3 - 375\lambda_1)/(\lambda_2(-5+7\sqrt{10Z^2+5-7Z})) + \frac{11}{15}(\sqrt{Z^2+1}\sqrt{10Z^2+5-7Z})\lambda_2^2 f(\xi)$$

$$p_4 = d_0 + \frac{11}{225}\sqrt{10Z^2+5-7Z}\,\lambda_2^3 g(\xi) + d_2 g(\xi)^2 + 2\sqrt{Z^2+1}\,d_2 f(\xi) + \frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})\sqrt{Z^2+1}f(\xi)g(\xi)$$

where

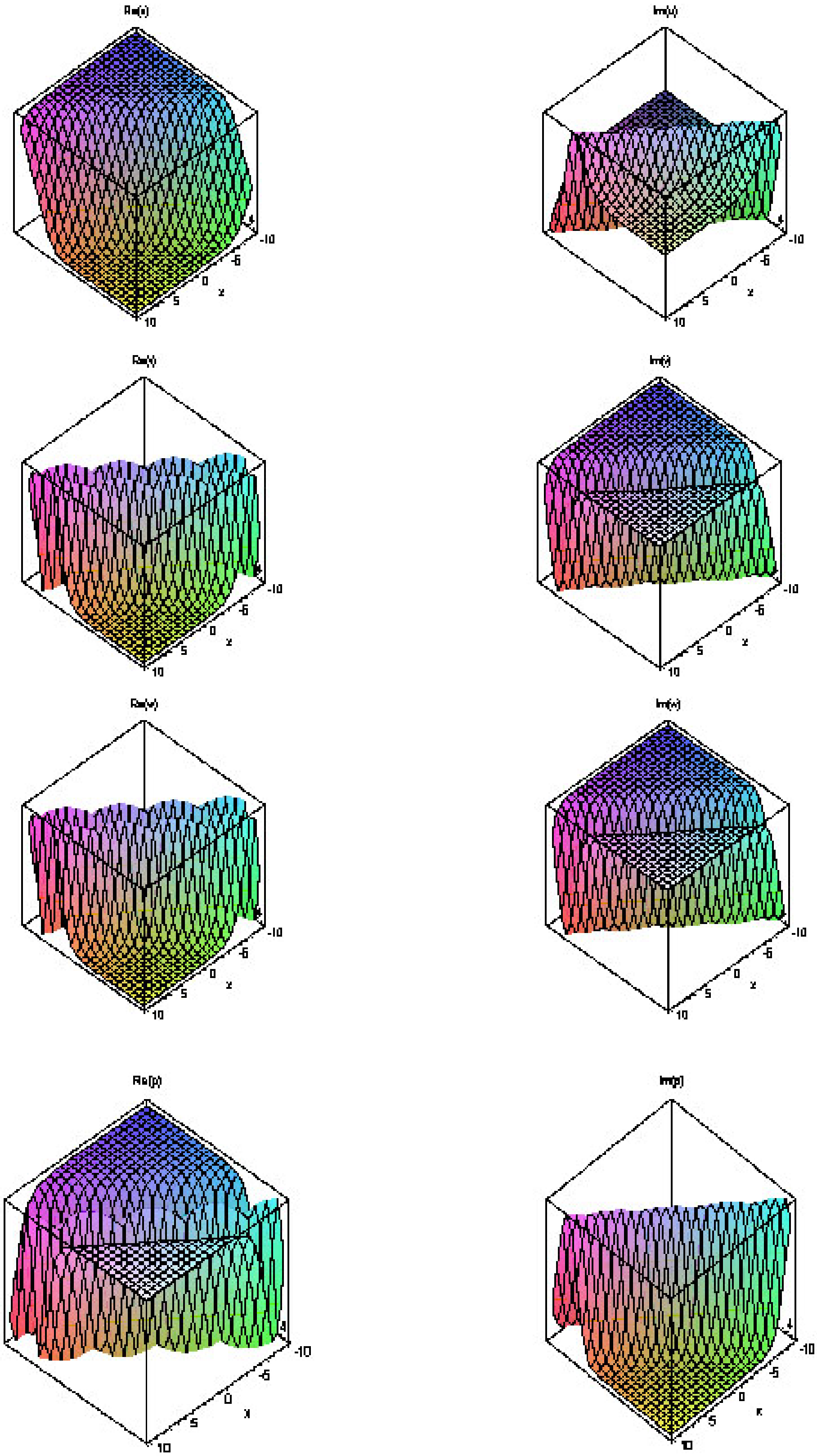
$$f(\xi) = \frac{1}{\sinh(\xi)+\sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)+\sqrt{Z^2+1}}$$

and $d_0,\ d_2,\ \lambda_1,\ \lambda_2,\ \lambda_3$ are arbitrary constants.

Case 5

$$u_5 = \sqrt{10Z^2+5-7Z}\,\lambda_2 g(\xi)$$

$$v_5 = \frac{11}{225}\lambda_2\lambda_3 + \frac{11}{15}\sqrt{10Z^2+5-7Z}\,\lambda_2\lambda_3\sqrt{Z^2+1}f(\xi)$$



$$w_5 = -\frac{1}{900}(210\sqrt{10Z^2+5-7Z}\,\lambda_2^3$$
$$-106\lambda_2^3 - 75\lambda_1 + 900\lambda_2 c_2)/\lambda_2 + c_2 g(\xi)^2$$
$$+\frac{1}{15}(11\lambda_2^2\sqrt{10Z^2+5-7Z} + 30c_2)\sqrt{Z^2+1}f(\xi)$$

$$p_5 = d_0 + \frac{11}{225}\sqrt{10Z^2+5-7Z}\,\lambda_2^3 g(\xi)$$
$$+d_2 g(\xi)^2 + 2\sqrt{Z^2+1}d_2 f(\xi)$$
$$+\frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})\sqrt{Z^2+1}f(\xi)g(\xi)$$

where

$$f(\xi) = \frac{1}{\sinh(\xi)+\sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)+\sqrt{Z^2+1}}$$

and d_0, d_2, λ_1, λ_2, λ_3 are arbitrary constants.

Case 6

$$u_6 = -\lambda_2 g(\xi)$$

$$v_6 = b_0 - b_0 g(\xi)^2 - 2\sqrt{Z^2+1}b_0 f(\xi)$$

$$w_6 = c_0 - \frac{1}{12}(12\lambda_2 c_0 + 4\lambda_2^3 - \lambda_1)g(\xi)^2/\lambda_2$$
$$-1/6\sqrt{Z^2+1}(12\lambda_2 c_0 + 4\lambda_2^3 - \lambda_1)f(\xi)/\lambda_2$$

$$p_6 = d_0 + d_2 g(\xi)^2 + 2\sqrt{Z^2+1}d_2 f(\xi)$$

where

$$f(\xi) = \frac{1}{\sinh(\xi)+\sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)+\sqrt{Z^2+1}}$$

and b_0, c_0, d_0, λ_1, λ_2, λ_3 are arbitrary constants.

Case 7

$$u_7 = \sqrt{10Z^2+5-7Z}\,\lambda_2 g_1(\xi)$$

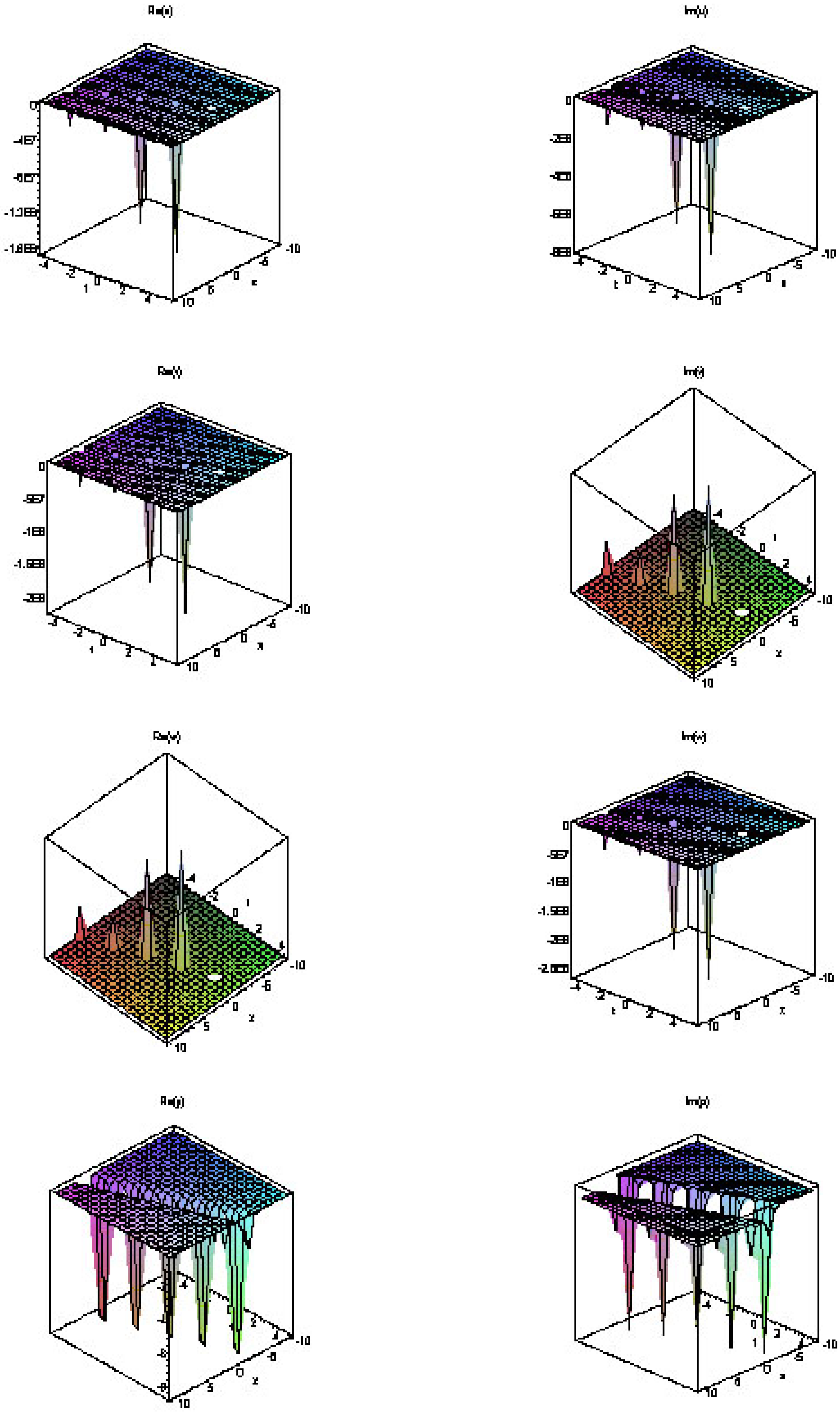
$$v_7 = b_0 + (\frac{11}{225}\lambda_2\lambda_3 - b_0)g(\xi)^2 + \frac{1}{18}(180b_0 - 55\lambda_2\lambda_3$$
$$-252\sqrt{10Z^2+5-7Z}\,b_0$$
$$+11\lambda_2\lambda_3\sqrt{10Z^2+5-7Z})$$
$$\sqrt{Z^2+1}f(\xi)/(-5+7\sqrt{10Z^2+5-7Z})$$

$$w_7 = c_0 - \frac{1}{900}(-3815\lambda_2^3 + 7391\sqrt{10Z^2+5-7Z}\,\lambda_2^3$$
$$+31500\lambda_2 c_0 + 900\sqrt{10Z^2+5-7Z}\,\lambda_2 c_0$$
$$-2625\lambda_1 - 75\sqrt{10Z^2+5-7Z}\,\lambda_1)$$
$$g(\xi)^2/(\lambda_2(35+\sqrt{10Z^2+5-7Z}))$$
$$+\frac{1}{90}(-190\lambda_2^3 + 900\lambda_2 c_0 - 75\lambda_1$$
$$+146\sqrt{10Z^2+5-7Z}\,\lambda_2^3$$
$$-1260\sqrt{10Z^2+5-7Z}\,\lambda_2 c_0$$
$$+105\sqrt{10Z^2+5-7Z}\,\lambda_1)\sqrt{Z^2+1}$$
$$f(\xi)/(\lambda_2(-5+7\sqrt{10Z^2+5-7Z}))$$

$$p_7 = d_0 + \frac{11}{225}\sqrt{10Z^2+5-7Z}\,\lambda_2^3 g(\xi)$$
$$+d_2 g(\xi)^2 + 2\sqrt{Z^2+1}d_2 f(\xi)$$
$$-\frac{11}{1500}\lambda_2^3(357\sqrt{10Z^2+5-7Z} - 5)$$
$$\sqrt{Z^2+1}f(\xi)g(\xi)/(-5+7\sqrt{10Z^2+5-7Z})$$

where

$$f(\xi) = \frac{1}{\sinh(\xi)+\sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)+\sqrt{Z^2+1}}$$



and $b_0, d_0, c_0, \lambda_1, \lambda_2, \lambda_3$ are arbitrary constants.

Case 8

$$u_8 = a_0 - \frac{1}{2}\lambda_2 g(\xi)$$

$$v_8 = b_0 - b_0 g(\xi)^2 - 2\sqrt{Z^2+1}b_0 f(\xi)$$

$$w_8 = c_0 - \frac{1}{12}(-\lambda_1 + \lambda_2^3 + 12\lambda_2 a_0^2 + 12\lambda_2 c_0)g(\xi)^2/\lambda_2 - \frac{1}{6}\sqrt{Z^2+1}(-\lambda_1 + \lambda_2^3 + 12\lambda_2 a_0^2 + 12\lambda_2 c_0)f(\xi)/\lambda_2$$

$$p_8 = d_0 + d_2 g(\xi)^2 + 2\sqrt{Z^2+1}d_2 f(\xi)$$

where

$$f(\xi) = \frac{1}{\sinh(\xi) + \sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi) + \sqrt{Z^2+1}}$$

and $a_0, b_0, c_0, d_0, \lambda_1, \lambda_2, \lambda_3$ are arbitrary constants.

2. when $\delta = -1$we get

Case 1

$$u_1 = \sqrt{10Z^2+5-7Z}\,\lambda_2 g(\xi)$$

$$v_1 = \frac{11}{225}\lambda_2\lambda_3 + \frac{11}{15}\sqrt{10Z^2+5-7Z}\,\lambda_2\lambda_3 f(\xi)$$

$$w_1 = \frac{1}{900}(-210\lambda_2^3\sqrt{10Z^2+5-7Z} + 106\lambda_2^3 + 75\lambda_1)/\lambda_2 + \frac{11}{15}\lambda_2^2\sqrt{10Z^2+5-7Z}\,f(\xi)$$

$$p_1 = d_0 + \frac{11}{225}\lambda_2^3\sqrt{10Z^2+5-7Z}\,g(\xi) + \frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})f(\xi)g(\xi)$$

where

$$f(\xi) = \frac{4}{5\cosh(\xi) + 3\sinh(\xi) + 1}$$
$$g(\xi) = \frac{5\sinh(\xi) + \cosh(\xi)}{5\cosh(\xi) + 3\sinh(\xi) + 1}$$

or

$$f(\xi) = \frac{1}{\cosh(\xi) + 1}$$
$$g(\xi) = \frac{\sinh(\xi)}{\cosh(\xi) + 1}$$

and $d_0, \lambda_1, \lambda_2, \lambda_3$ are arbitrary constants.

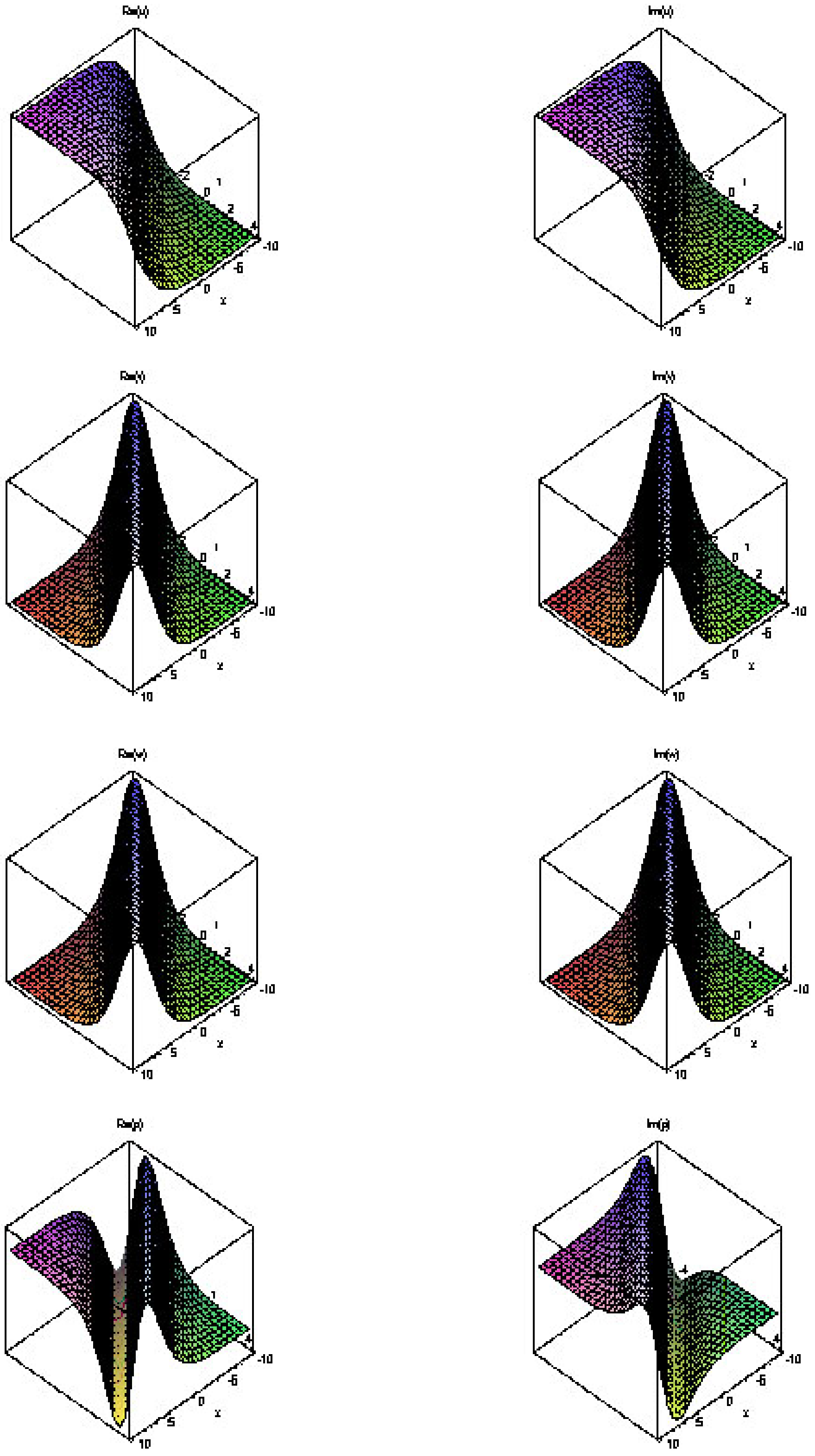
Case 2

$$u_2 = \sqrt{10Z^2+5-7Z}\,\lambda_2 g(\xi)$$

$$v_2 = \frac{11}{225}\lambda_2\lambda_3 - \frac{11}{15}\sqrt{10Z^2+5-7Z}\,\lambda_2\lambda_3 f(\xi)$$

$$w_2 = \frac{1}{900}(-210\lambda_2^3\sqrt{10Z^2+5-7Z} + 106\lambda_2^3 + 75\lambda_1)/\lambda_2 - \frac{11}{15}\lambda_2^2\sqrt{10Z^2+5-7Z}\,f(\xi)$$

$$p_2 = d_0 + \frac{11}{225}\lambda_2^3\sqrt{10Z^2+5-7Z}\,g(\xi) - \frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})f(\xi)g(\xi)$$



where

$$f(\xi)=\frac{4}{5\cosh(\xi)+3\sinh(\xi)-1}$$
$$g(\xi)=\frac{5\sinh(\xi)+\cosh(\xi)}{5\cosh(\xi)+3\sinh(\xi)-1}$$

or

$$f(\xi)=\frac{1}{\cosh(\xi)-1}$$
$$g(\xi)=\frac{\sinh(\xi)}{\cosh(\xi)-1}$$

and $d_0,\ \lambda_1,\ \lambda_2,\ \lambda_3$ are arbitrary constants

Case 3

$$u_3=\sqrt{10Z^2+5-7Z}\,\lambda_2 g(\xi)$$

$$v_3=\frac{11}{225}\lambda_2\lambda_3+\frac{11}{15}\sqrt{10Z^2+5-7Z}\,\lambda_2\lambda_3 f(\xi)$$

$$w_3=\frac{1}{900}(-210\lambda_2^3\sqrt{10Z^2+5-7Z}$$
$$+106\lambda_2^3+75\lambda_1)/\lambda_2$$
$$+\frac{11}{15}\lambda_2^2\sqrt{10Z^2+5-7Z}\,f(\xi)$$

$$p_3=d_0+\frac{11}{225}\lambda_2^3\sqrt{10Z^2+5-7Z}\,g(\xi)$$
$$+d_2 g(\xi)^2+2d_2 f(\xi)$$
$$+\frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})f(\xi)g(\xi)$$

where

$$f(\xi)=\frac{4}{5\cosh(\xi)+3\sinh(\xi)+1}$$
$$g(\xi)=\frac{5\sinh(\xi)+\cosh(\xi)}{5\cosh(\xi)+3\sinh(\xi)+1}$$

or

$$f(\xi)=\frac{1}{\cosh(\xi)+1}$$
$$g(\xi)=\frac{\sinh(\xi)}{\cosh(\xi)+1}$$

and $d_0,\ d_2,\ \lambda_1,\ \lambda_2,\ \lambda_3$ are arbitrary constants

Case 4

$$u_4=\sqrt{10Z^2+5-7Z}\,\lambda_2 g(\xi)$$

$$v_4=\frac{11}{225}\lambda_2\lambda_3-\frac{11}{15}\sqrt{10Z^2+5-7Z}\,\lambda_2\lambda_3 f(\xi)$$

$$w_4=\frac{1}{900}(-210\lambda_2^3\sqrt{10Z^2+5-7Z}$$
$$+106\lambda_2^3+75\lambda_1)/\lambda_2-\frac{11}{15}\lambda_2^2$$
$$\sqrt{10Z^2+5-7Z}\,f(\xi)$$

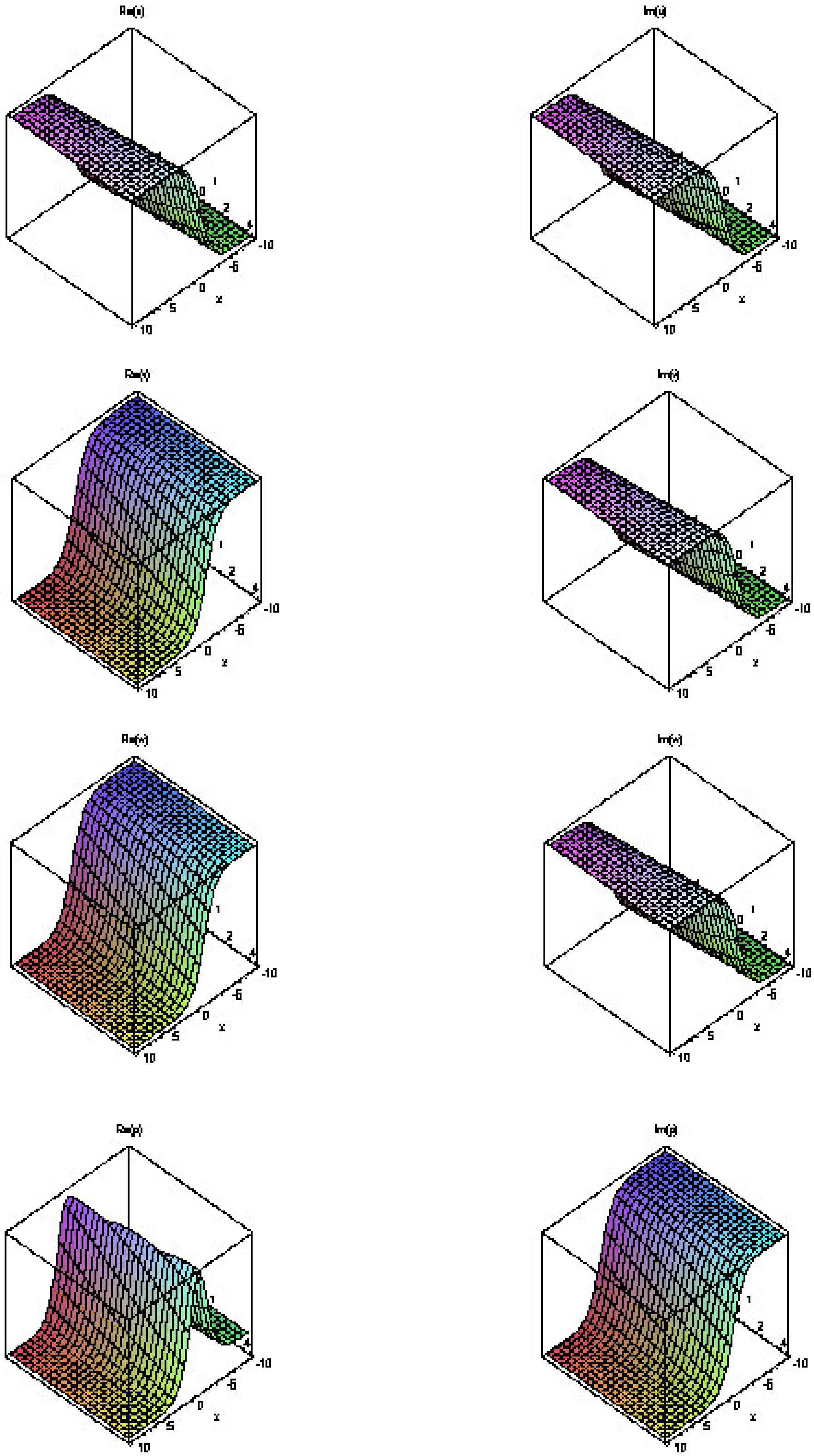
$$p_4=d_0+\frac{11}{225}\lambda_2^3\sqrt{10Z^2+5-7Z}\,g(\xi)$$
$$+d_2 g(\xi)^2-2d_2 f(\xi)-\frac{11}{150}\lambda_2^3$$
$$(-5+7\sqrt{10Z^2+5-7Z})f(\xi)g(\xi)$$

where

$$f(\xi)=\frac{4}{5\cosh(\xi)+3\sinh(\xi)-1}$$
$$g(\xi)=\frac{5\sinh(\xi)+\cosh(\xi)}{5\cosh(\xi)+3\sinh(\xi)-1}$$

or

$$f(\xi)=\frac{1}{\cosh(\xi)-1}$$
$$g(\xi)=\frac{\sinh(\xi)}{\cosh(\xi)-1}$$


Re(u)
Im(u)
Re(v)
Im(v)
Re(w)
Im(w)
Re(p)
Im(p)

and d_0, d_2, λ_1, λ_2, λ_3 are arbitrary constants.

Case 5

$$u_5 = \frac{1}{9}\lambda_2(7\sqrt{3Z^2+2-Z}-26)/(\sqrt{3Z^2+5}+8\sqrt{3Z^2+2-Z})(-2+\sqrt{3Z^2+2-Z})+\sqrt{3Z^2+2-Z}\lambda_2 g(\xi)$$

$$v_5 = \frac{1}{18}\lambda_2\lambda_3(29\sqrt{3Z^2+2-Z}-73)/(\sqrt{3Z^2+2-Z}+4)+\frac{1}{3}\sqrt{3Z^2+5+8\sqrt{3Z^2+2-Z}}\lambda_3\lambda_2 g(\xi)$$

$$w_5 = \frac{1}{36}(-3461656\sqrt{3Z^2+2-Z}\lambda_2^3+66993\sqrt{3Z^2+2-Z}\lambda_1+1437968\lambda_2^3+136914\lambda_1)/(\lambda_2(22331\sqrt{3Z^2+2-Z}+45638))+\frac{1}{27}\lambda_2^2(463\sqrt{3Z^2+2-Z}+502)g(\xi)/\sqrt{3Z^2+5+8\sqrt{3Z^2+2-Z}}(7\sqrt{3Z^2+2-Z}-26)$$

$$p_5 = d_0+\frac{1}{27}\lambda_2^3(401\sqrt{3Z^2+2-Z}+38)g(\xi)/(7\sqrt{3Z^2+2-Z}-26)+\frac{1}{3}\lambda_2^3\sqrt{3Z^2+5+8\sqrt{3Z^2+2-Z}}\sqrt{3Z^2+2-Z}g(\xi)^2$$

where

$$f(\xi) = \frac{4}{5\cosh(\xi)+3\sinh(\xi)}$$

$$g(\xi) = \frac{5\sinh(\xi)+\cosh(\xi)}{5\cosh(\xi)+3\sinh(\xi)}$$

or

$$f(\xi) = \frac{1}{\cosh(\xi)}$$

$$g(\xi) = \frac{\sinh(\xi)}{\cosh(\xi)}$$

and d_0, λ_1, λ_2, λ_3 are arbitrary constants.

Case 6

$$u_6 = \sqrt{10Z^2+5-7Z}\lambda_2 g(\xi)$$

$$v_6 = \frac{11}{225}\lambda_2\lambda_3+\frac{11}{15}\sqrt{10Z^2+5-7Z}\lambda_2\lambda_3 f(\xi)$$

$$w_6 = \frac{1}{900}(-210\lambda_2^3\sqrt{10Z^2+5-7Z}+106\lambda_2^3+75\lambda_1-900\lambda_2 c_2)/\lambda_2+c_2 g(\xi)^2+(\frac{11}{15}\lambda_2^2\sqrt{10Z^2+5-7Z}+2c_2)f(\xi)$$

$$p_6 = d_0+\frac{11}{225}\lambda_2^3\sqrt{10Z^2+5-7Z}g(\xi)+\frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})f(\xi)g(\xi)$$

where

$$f(\xi) = \frac{4}{5\cosh(\xi)+3\sinh(\xi)+1}$$

$$g(\xi) = \frac{5\sinh(\xi)+\cosh(\xi)}{5\cosh(\xi)+3\sinh(\xi)+1}$$

or

$$f(\xi) = \frac{1}{\cosh(\xi)+1}$$

$$g(\xi) = \frac{\sinh(\xi)}{\cosh(\xi)+1}$$

and d_0, c_2, λ_1, λ_2, λ_3 are arbitrary constants

Case 7

$$u_7 = \sqrt{10Z^2+5-7Z}\lambda_2 g(\xi)$$

$$v_7 = \frac{11}{225}\lambda_2\lambda_3 - \frac{11}{15}\sqrt{10Z^2+5-7Z}\lambda_2\lambda_3 f(\xi)$$

$$w_7 = \frac{1}{900}(-210\lambda_2^3\sqrt{10Z^2+5-7Z} + 106\lambda_2^3 + 75\lambda_1 - 900\lambda_2 c_2)/\lambda_2 + c_2 g(\xi)^2 + (-\frac{11}{15}\lambda_2^2\sqrt{10Z^2+5-7Z} - 2c_2)f(\xi)$$

$$p_7 = d_0 + \frac{11}{225}\lambda_2^3\sqrt{10Z^2+5-7Z}g(\xi) - \frac{11}{150}\lambda_2^3(-5+7\sqrt{10Z^2+5-7Z})f(\xi)g(\xi)$$

where

$$f(\xi) = \frac{4}{5\cosh(\xi)+3\sinh(\xi)-1}$$
$$g(\xi) = \frac{5\sinh(\xi)+\cosh(\xi)}{5\cosh(\xi)+3\sinh(\xi)-1}$$

or

$$f(\xi) = \frac{1}{\cosh(\xi)-1}$$
$$g(\xi) = \frac{\sinh(\xi)}{\cosh(\xi)-1}$$

and d_0, c_2, λ_1, λ_2, λ_3 are arbitrary constants

Solution of some of the cases
case 6, with

$$f(\xi) = \frac{1}{\sinh(\xi)+\sqrt{Z^2+1}}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)+\sqrt{Z^2+1}}$$

and when

$y = 1.5;\ b_0 = 0.2;\ c_0 = -0.1;\ d_0 = 0.4;\ d_2 = 0.5;$
$\lambda_1 = 1.2;\ \lambda_2 = -0.5;\ \lambda_3 = 0.33;$

Case 1, with

$$f(\xi) = \frac{1}{\sinh(\xi)}$$
$$g(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}$$

and when

$y = 0.5;\ d_0 = -0.3;\ \lambda_1 = 0.4;\ \lambda_2 = -0.5;\ \lambda_3 = 1;$

Case 3, with δ= -1 and

$$f(\xi) = \frac{1}{\cosh(\xi)+1}$$
$$g(\xi) = \frac{\sinh(\xi)}{\cosh(\xi)+1}$$

and when

$y = 0.3;\ d_0 = 1;\ d_2 = -0.5;\ \lambda_1 = 1;\ \lambda_2 = 0.5;\ \lambda_3 = 0.8;$

Case 5, with δ = -1 and

$$f(\xi) = \frac{4}{5\cosh(\xi)+3\sinh(\xi)}$$
$$g(\xi) = \frac{5\sinh(\xi)+\cosh(\xi)}{5\cosh(\xi)+3\sinh(\xi)}$$

and when

$y = 1;\ d_0 = -1.5;\ \lambda_1 = 0.1;\ \lambda_2 = -0.5;\ \lambda_3 = -1;$

CONCLUSION

In this paper, based on a modification of a homogeneous balance method, we have proposed the extended Riccati equation and applied it to find many exact soliton solutions for the (2+1)-dimensional higher order Broer-Kaup system (1). In particular, some new exact solitary wave solutions are given.

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