Compton scattering - A simplified derivation

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ABSTRACT

The general result of the Compton process is derived in this paper without the assumption that one of the particles should be initially at rest. Nor is it assumed that one of the particles is a photon.

Key words: A photon, Compton scattering, derivation.

INTRODUCTION

(E_1), and $(\overrightarrow{dp_2})$ by $(\overrightarrow{-dp_1})$, $E_1 dE_1 = c^2 \overrightarrow{p_1}.\overrightarrow{dp_1}$ $(\overrightarrow{dp_1} + \overrightarrow{dp_2} = 0)$

Any text-book on physical chemistry (or physics) with chapters on quantum mechanics introduces wave particle dualism through discussions supported by descriptions of experimental work. The non-coherent scattering of photons by weakly-bound(almost free) electrons known as Compton scattering always finds a place in the discussion with equations for momentum and energy conservation, to derive the relation.

$$(\lambda - \lambda_0) = (\frac{h}{m_0 c})(1 - \cos\theta)$$

Quite a number of derivations[1-4] of the result are available in literature. But an extremely simple and short derivation that brings out the conceptual essence of a relativistic two-particle collision is worth an examination. This is presented here in its most general form and the Compton process is shown as a special case of it.

Derivation

The relativistic dependence of mass-energy on momentum is given by

$$E^2 = p^2 c^2 + (m_0 c^2)^2 \qquad ...(1)$$

In a two particle elastic collision, the total energy $E = (E_1 + E_2)$ and the total momentum vector $\vec{p} = \overrightarrow{p_1} + \overrightarrow{p_2}$ are both conserved. Taking differentials of equations (1) for the particles

and $E_2 dE_2 = c^2 \overrightarrow{p_2}.\overrightarrow{dp_2}$ ...(2)

Since and $\vec{p}$ are constant,

$(dE_1 + dE_2 = 0)$ and

Thus by replacing dE_2 by

We get from (2) by subtraction

$$(E_1 + E_2) dE_1 = c^2 (\overrightarrow{p_1} + \overrightarrow{p_2}).\overrightarrow{dp_1}$$

i.e.

$$E dE_1 = c^2 \overrightarrow{p}.\overrightarrow{dp_1} \qquad ...(3)$$

Essentially this is Compton formula in the differential form.

If we indicate by (') and ('') the initial and final situations in a two-particle elastic collision, then by subtraction

$$Ed(E_1' - E_1'') = c^2 \overline{p} d(\overline{p_1}' - \overline{p_1}'') \qquad ...(3)$$

This on integration gives

$$E(E_1' - E_1'') = c^2 \overline{p}.(\overline{p_1}' - \overline{p_1}'') \qquad ..(4)$$

Observe that no arbitrary constant is needed since when i.e. no transfer of energy and momentum (equivalent to no collision at all).

Then for the photon collision with a (nearly) stationary electron.

$$(h\nu_0 + m_0c^2)(h\nu_0 - h\nu)$$
$$= c^2\left(\frac{\overline{h\nu_0}}{c}\right).\left(\frac{\overline{h\nu_0}}{c} - \frac{\overline{h\nu}}{c}\right) = (\overline{h\nu_0}).(\overline{h\nu_0} - h\nu)$$

This by easy algebra gives

$$m_0c^2(h\nu_0 - h\nu) = (h\nu_0)(h\nu)\cos\theta$$

$$\therefore c\left(\frac{\nu_0 - \nu}{\nu\nu_0}\right) = \frac{c}{\nu} - \frac{c}{\nu_0} = (\lambda - \lambda_0) = \left(\frac{h}{m_0 c}\right)(1 - \cos\theta) \qquad ...(5)$$

CONCLUSION

In the derivation, separate equations are not written for momentum and energy conservation. Only one relativistic equation viz $E^2 = p^2C^2 + m_o^2 C^4$, relating energy and momentum is considered. This is shown to be conceptually sufficient for deriving the general result.

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