Violation of Third Generalization Bell Inequality in Quantum Theory

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ABSTRACT

This paper we investigated on quantum theory in detail about subjects such as causality, locality and hidden variable, then assuming variables of bell inequality locality-causality. As a result, achieved to quantum theory and by use of entanglement state applied some examples as a sample for Violation of third generalization bell inequality in quantum theory.

Key words: quantum theory, causality, locality, hidden variable, entanglement state, Bell inequality.

INTRODUCITON

In order to find out a more general prove for Bell thesis at least we have to, tell some essential imagination of locality-causality hidden variable for experimental Bohm [1,2]. After disport two particles, device should have defined the result measurement the spin component for each particle. This virtue had been made despite real spin, but general it 's not necessary that the hidden variable follow an special model [3,4].

RESULTS AND DISCUSSION

Third generalization of bell inequality

The base of this inequality which is famous to Clauser and Shimony is same to former. Means that this inequality works based on the average amount of appoint results. So it's expected value is like the following [5-7].

$$C(a,b)=\int d\lambda_{HV}\,\rho(\lambda_{HV})\overline{A}_a(\lambda_{HV})\overline{B}_b(\lambda_{HV}) \qquad (1)$$

$$A_a(\lambda_{HV})=\pm 1 \rightarrow \left|\overline{A}_a\right| \leq 1$$

$$B_b(\lambda_{HV})=\pm 1 \rightarrow \left|\overline{B}_b\right| \leq 1$$

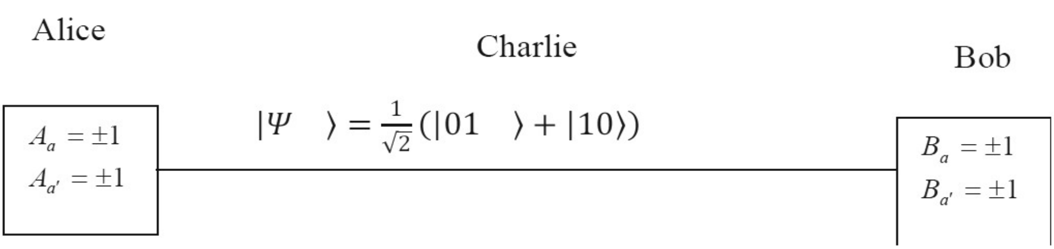

Fig. 1 : Generalization of bell inequality

Charlie provides the interlaced qubit for bob and Alice. And send it to them. Bob and Alice can appoint on interlaced qubit which are sent to them. They choice base measurement a,a' for b' b, for Alice and they measure random, for example device eshternger lakh. Some measurement will be done on the system. Singlet state quantum which are on a single state quantum means;

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle\right) \qquad (2)$$

Assume $A_a \Rightarrow \frac{2}{h}\vec{a}.\vec{s}\ if\ \vec{a}.\vec{s} = \pm\frac{h}{2} \Rightarrow \pm 1$ for up spin and (-) for down spin

Then assume $B_b \Rightarrow \frac{2}{h}\vec{a}.\vec{s}\ if\ \vec{a}.\vec{s} = \pm\frac{h}{2} \Rightarrow \pm 1$ for up spin and (-) for down spin

$$\begin{aligned}
&|C(a,b) - C(a,b')| \le \\
&= \int d\lambda_{HV}\,\rho(\lambda_{HV})[1 \pm \bar{A}_{a'}(\lambda_{HV})\bar{B}_{b'}(\lambda_{HV})] + \int d\lambda_{HV}\,\rho(\lambda_{HV})[1 \pm \bar{A}_{a'}(\lambda_{HV})\bar{B}_{b}(\lambda_{HV})] \\
&\Rightarrow |C(a,b) - C(a,b')| \le 1 \pm \int d\lambda_{HV}\,\rho(\lambda_{HV})\bar{A}_{a'}(\lambda_{HV})\bar{B}_{b'}(\lambda_{HV}) \\
&1 \pm \int d\lambda_{HV}\,\rho(\lambda_{HV})\bar{A}_{a'}(\lambda_{HV})\bar{B}_{b}(\lambda_{HV}) \Rightarrow |C(a,b) - C(a,b')| \le 2 \pm (C(a',b') + C(a',b)) \\
&\Rightarrow |C(a,b) - C(a,b')| + |(C(a',b') + C(a',b))| \le 2 \qquad (3)
\end{aligned}$$

Example for violation of third generalization bell theory easily we can make an example in mechanic quantum theory (Clauser and Shimony). But first we find out the expected value of $[C(a,b)]_{QM}$ in mechanic quantum theory again in same way by using above contexts. So

$$C(a,b) = \left(\frac{2}{\hbar}\right)^2_{\psi} \langle \vec{a}\cdot\vec{s}_1 \otimes \vec{a}\cdot\vec{s}_2 \rangle_{\psi} \qquad (4)$$

To calculate this correlation we imaging

So we have

$$\vec{s}_1 = \frac{\hbar}{2}(\sigma_{1x} + \sigma_{1y} + \sigma_{1z})$$

$$(5)$$

$$\vec{s}_2 = \frac{\hbar}{2}(\sigma_{2x} + \sigma_{2y} + \sigma_{2z})$$

$$\vec{a}\cdot\hat{s}_1 = \frac{\hbar}{2}a_z\sigma_{1z}$$

$$\vec{b}\cdot\hat{s}_2 = \frac{\hbar}{2}(b_x\sigma_{2x} + b_z\sigma_{2z}) = \frac{\hbar}{2}(\sigma_{2x}\sin\theta_{ab} + \sigma_{2z}\cos\theta_{ab})$$

$$C(\vec{a},\vec{b}) = \langle\psi|\sigma_{1z} \otimes (\sigma_{2x}\sin\theta_{ab} + \sigma_{2z}\cos\theta_{ab})|\psi\rangle \qquad ...(6)$$

$$\sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_{2X}Sin\theta_{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} Sin\theta_{ab} = \begin{pmatrix} 0 & Sin\theta_{ab} \\ Sin\theta_{ab} & 0 \end{pmatrix}, \quad .... \qquad ...(7)$$

$$\sigma_{1z} \otimes (\sigma_{2x}\sin\theta_{ab} + \sigma_{2z}\cos\theta_{ab}) =$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} \cos\theta_{ab} & \sin\theta_{ab} \\ \sin\theta_{ab} & -\cos\theta_{ab} \end{pmatrix} =$$

$$\begin{pmatrix} \cos\theta_{ab} & \sin\theta_{ab} & 0 & 0 \\ \sin\theta_{ab} & -\cos\theta_{ab} & 0 & 0 \\ 0 & 0 & -\cos\theta_{ab} & -\sin\theta_{ab} \\ 0 & 0 & -\sin\theta_{ab} & \cos\theta_{ab} \end{pmatrix} \qquad ...(8)$$

By using this relation

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle) \quad |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}] =$$

$$\frac{1}{\sqrt{2}}[\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}] = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow \langle\psi| = \frac{1}{\sqrt{2}}(0 \quad 1 \quad -1 \quad 0) \qquad ...(9)$$

So

$$\langle\psi|\sigma_{1z} \otimes (\sigma_{2x}\sin\theta_{ab} + \sigma_{2z}\cos\theta_{ab})|\psi\rangle = ?$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}] =$$

$$\frac{1}{\sqrt{2}}[\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}] = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow \langle\psi| = \frac{1}{\sqrt{2}}(0 \quad 1 \quad -1 \quad 0)$$

Similarity

$$C(\vec{a},\vec{b}') = -Cos\theta_{ab'}, \quad C(\vec{a}',\vec{b}) = -Cos\theta_{a'b}, \quad C(\vec{a}',\vec{b}') = -Cos\theta_{a'b'}$$

So

$$|Cos\theta_{ab} - Cos\theta_{ab'} + Cos\theta_{a'b} + Cos\theta_{a'b'}| \le 2 \qquad (9)$$

Now we assume which Charlie sent state entanglement For Bob and Alice:

$$|\Psi\square\rangle = \frac{1}{\sqrt{2}}(|01\square\rangle + |10\rangle)$$

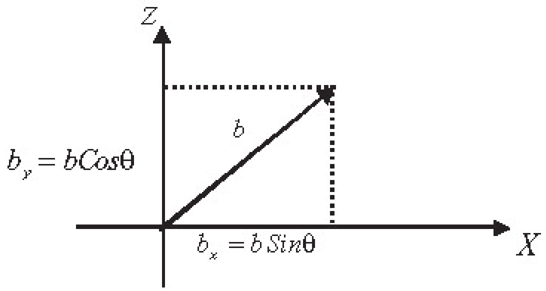

Fig. 1:

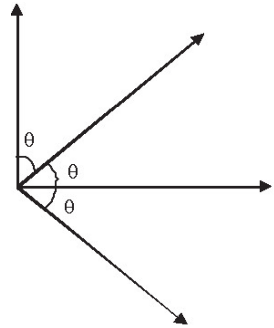

Fig. 2: Choose base measurement a, a, b, b

They measure on base $b,b',a\,a'\ \text{and}\ \theta = \frac{\Pi}{4}$

By a simple calculating, we can show that in the relation (2) for clauser and shimony inequality in quantum mechanics is equal to:

$$S = \left|C(a,b) - C(a,b') + C(a',b') + C(a',b)\right|_{QM}$$

$$C(a,b) = -\cos\ \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$C(a,b') = -\cos\ \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

$$C(a',b') = -\cos\ \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$C(a',b) = -\cos\ (-\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$$

$$\left|C(a,b) - C(a,b') + C(a',b') + C(a',b)\right|_{QM}$$

$$\left|-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right|_{QM} = 2\sqrt{2} \qquad (12)$$

Violation example of clauser and shimony inequality in lab

Bell and others with a kind of examination which parameter named S showed that it's able to differentiate between mechanic quantum and these last variable theory. Local theory privies S is always smaller than 2. Just the same in mechanic quantum privies, S=sqr(2)*2 which S is bigger than 2. So bell theory has been violated.

An examination by accelerator, electron, and position set clashed and made their Mesons and antiparticle which crack up to lighter parts. A couple mesons behave like a couple photons. But Belle group analyzed the correlation of particle-antiparticle through away named, instead of analyzing the correlation of polarity sights . And find the amount S=2.725. in describing basic nature So quantum mechanics seems between un locality correlation particle.

CONCLUSIONS

In Bell inequality examination, the couple virtues of parts which in special relativity their distance is space are calculated. Mechanic quantum predicts that it could be a unlocality correlation among particle. But some physiques believe it couldn't be right and quantum particle should have local late them (named hidden variable). It should be attended if mechanics quantum for sights violent bell inequality excrementally, we have to accept of non local as a basic natural identity, non local are one of basic sights nature and every theory which want to claim to introduce quantum nature should be non locality.

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