Electromagnetic Wave Propagation Through Triangular Antenna Subject to Longitudinal Symmetric Conditions

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ABSTRACT

Electromagnetic (EM) field intensities happen to exist as solutions of Maxwell's equations in a three dimensional space. In the present paper an attempt has been made to determine the components of EM field intensities belonging to a pair of groove regions adjacent to a convex triangular prism. Field intensities are supposed to be longitudinal symmetry and the triangular prism forms a part of an echellete grating of fixed period. Two existence theorem regarding longitudinal symmetric electric intensity vector and magnetic intensity vector associated with time dependent spherical wave have been established. Finally, the expression for the current density associated with the prism has been derived.

Key words: Electromagnetic field intensities, Convex triangular prism, Maxwell's equations.

INTRODUCTION

A convex triangular obstacle forms a vital part of a periodic echellete grating. In recent years[1-10] quite a good number of results have been reported pertaining to the reflection, grazing and the diffraction of a propagating EM wave through a smooth, conducting and convex triangular obstacle K (Fig. 1). The obstacle is supposed to be hollow with an open rectangular base having the flare angle ², the groove depth 'h' and the period 'd' associated with the said echellete grating. In the present paper a model M (Fig. 2), consisting of a convex triangular obstacle and its adjacent wedge regions R_i (i = 1, 2), has been considered for interacting with a propagating EM wave. EM field intensities $F = (H \vee E)$ are generated due to propagating EM wave subject to governing Maxwell's equations

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} = \sigma \boldsymbol{E} + \in \frac{\partial \boldsymbol{E}}{\partial t}, \quad \nabla \times \boldsymbol{E} = -\frac{\partial \hat{\boldsymbol{A}}}{\partial t} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$

and

$$\nabla^2 \boldsymbol{F} = \mu \left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2} \right)$$

where H, E and $\boldsymbol{F} = (\boldsymbol{H} \vee \boldsymbol{E})$ being magnetic intensity vector, electric intensity vector and their combination respectively. The physical elements Ã,Î, , J and B stand for conductivity, permittivity, permeability, current density and magnetic flux density associated with the model. The object of present paper is to determine the vector field intensity F by using spherical polar coordinates (r, ¸, f) subject to the assumption $\frac{\partial \boldsymbol{F}}{\partial \phi} = 0$ (Longitudinal symmetric condition), leading finally to a spherical wave. Two existence theorems have been established for finding the components of $\boldsymbol{H}$ and $\boldsymbol{E}$ is associated with the Maxwell's equations in $\boldsymbol{H}$ and $\boldsymbol{E}$ justifying thereby the transmission ($\sigma \neq 0$) of the concerning EM waves through K. The result has been further utilized for computing the current density $\boldsymbol{J}$.

FORMULATION OF THE PROBLEM

Consider the Maxwell's equation

$$\nabla^2 \boldsymbol{F} = \frac{\partial^2 \boldsymbol{F}}{\partial x_1^2} + \frac{\partial^2 \boldsymbol{F}}{\partial x_2^2} + \frac{\partial^2 \boldsymbol{F}}{\partial x_3^2} = \mu \left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2} \right) \quad ...(1)$$

where $\boldsymbol{F} = \boldsymbol{F}(x_1, x_2, x_3, t)$ stands for vector field intensity. Transforming (1) by using spherical polar coordinates $x_1 = r \sin\theta \cos\phi$, $x_2 = r \sin\theta \sin\phi$,

$x_3 = r\cos\theta$ one can arrive at the equation

$$\nabla^2 \boldsymbol{F} = \frac{1}{r^2\sin\theta}\left[r^2\sin\theta\frac{\partial^2\boldsymbol{F}}{\partial r^2} + 2r\sin\theta\frac{\partial\boldsymbol{F}}{\partial r} + \sin\theta\frac{\partial^2\boldsymbol{F}}{\partial\theta^2}\right. \quad \text{...(2)}$$

$$\left. + \cos\theta\frac{\partial\boldsymbol{F}}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial^2\boldsymbol{F}}{\partial\phi^2}\right] = \mu\left(\sigma\frac{\partial\boldsymbol{F}}{\partial t} + \in\frac{\partial^2\boldsymbol{F}}{\partial t^2}\right)$$

Now, using the variable separable method, one can arrive at the solution of the equation (2) in the form

$$\boldsymbol{F}(r,\theta,\phi,t) = F_1(r)F_2(\theta,\phi)G(t) \quad \text{...(3)}$$

where $F_1(r)$, $F_2(\theta,\phi)$ and $G(t)$ satisfy the equations

$$\frac{1}{r^2}\left[\frac{r^2F_1''}{F_1} + \frac{2rF_1'}{F_1} + \frac{1}{F_2}\frac{\partial^2F_2}{\partial\theta^2} + \cot\theta\frac{1}{F_2}\frac{\partial F_2}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{1}{F_2}\frac{\partial^2F_2}{\partial\phi^2}\right] \quad \text{...(4)}$$

$$= \frac{\mu(\sigma G'(t) + \in G''(t))}{G} = -k^2$$

and

$$\frac{r^2F_1''}{F_1} + \frac{2rF_1'}{F_1} + k^2r^2 = -\left[\frac{1}{F_2}\frac{\partial^2F_2}{\partial\theta^2} + \cot\theta\frac{1}{F_2}\frac{\partial F_2}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{1}{F_2}\frac{\partial^2F_2}{\partial\phi^2}\right] = \xi \quad \text{...(5)}$$

where the constants *k* and ξ are independent of *r*, ¸, φ and *t*.

In particular, assuming $\xi = n(n+1) \forall n \in J^+$ then equation (5) give arise to the surface harmonic function F_2(¸, *f*) satisfying the equation

$$\frac{\partial^2F_2}{\partial\theta^2} + \cot\theta\frac{\partial F_2}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2F_2}{\partial\phi^2} + n(n+1)F_2 = 0 \quad \text{...(6)}$$

Again, separating 'F_2' further in the form of the product

$$F_2(\theta,\phi) = F_3(\theta)F_4(\phi) \quad \text{...(7)}$$

one can arrive at the equations

$$F_4'' + m^2F_4 = 0 \quad \text{...(8)}$$

and

$$(1-x^2)F_3'' - 2xF_3' + \left[n(n+1) - \frac{m^2}{1-x^2}\right]F_3 = 0 \quad \text{... (9)}$$

where , $x = \cos\theta$, *m* is independent of ¸.

The equation (9) may be identified as associated Legendre's equation which furnishes the associated Legendre's function $P_n^m(x) = F_3(x)$ as one of its solutions. Thus, combining (7), (8) and (9), the surface harmonic function F_2(¸, *f*) may be expressed in the form

$$F_2(\theta,\phi) = C_3P_n^m(x)e^{Jm\phi}\left(J = \sqrt{-1}\right) \quad \text{...(10)}$$

where C_3 is an arbitrary constant.

Now, introducing longitudinal symmetry condition $\frac{\partial\mathbf{F}}{\partial\phi} = 0$ for the EM field ***F***(*r*, ¸, *f*, *t*) the equation (9) apparently reduces to the Legendre's equation

$$(1-x^2)\frac{d^2F_3}{dx^2} - 2x\frac{dF_3}{dx} + n(n+1)F_3 = 0 \quad \text{... (11)}$$

furnishing thereby the Legendre polynomial $P_n(x)$ as one of its solutions, and consequently the solution (10) reduces to the form

$$F_2(\theta,\phi) = C_3P_n(x) \quad \text{... (12)}$$

Now, recalling the equation (5) again, one can arrive at the ordinary differential equation

$$r^2F_1'' + 2rF_1' + k^2r^2F_1 - n(n+1)F_1 = 0 \quad \text{...(13)}$$

which possesses the only regular singular point at r = 0 and as such applying Frobenius method one can arrive at the series solution of (13) in the form

$$F_1(r) = \sum_{m=0}^{\infty} a_{2m}r^{2m+p} \quad \text{...(14)}$$

which converge within a sphere $|r| < K, (\forall K \in R)$ for

$$2p = -1 \pm \sqrt{1 + 4n(n+1)} \quad \text{...(15)}$$

and the coefficients a_{2m} satisfy the recurrence relation

$$a_{2m} = -k^2 a_2(m-1)/[(p+2m)(p+2m+1)-n(n+1)](\forall m \in J^+)$$ ...(16)

Again, referring to the equation (4), one can arrive at the equation

$$\mu \in G''(t) + \mu\sigma G'(t) + k^2 = 0$$ ... (17)

which furnishes the solution

$$G(t) = A \exp\left\{ j\omega - \left(\frac{\sigma}{2\in} \right) \right\} t$$ ...(18)

where 'A' is an arbitrary constant, $j = \sqrt{-1}$, and

$$2\mu \in \omega = \rho = \sqrt{-\mu^2\sigma^2 + 4\mu \in k^2}$$

and k is the wave number restricted by the inequality

$$2k\sqrt{\in} > \sqrt{\mu}\,\sigma$$ ...(19)

Hence, combining (3), (12), (16) and (18) one can arrive at the solution

$$\boldsymbol{F}(r,\theta,t) = \sum_{n=1,m=0}^{\infty} A_n a_{2m} r^{2m+p} P_n(\cos\theta)\exp[j\omega - (\sigma/2\in)]t$$ ...(20)

Spherical wave function and the components of electric and magnetic intensities vectors :

The expression (20) represent a spherical wave function

$$\Phi^F(r,\theta,t) = \Phi^F(r,\theta)e^{-\sigma t/2\in}e^{j\omega t}$$ ...(21)

where $\Phi^F(r,\theta) = \sum_{n=1}^{\infty} A_n F_1(r) P_n(\cos\theta)$ stands for the free space spherical wave formed by superimposition of spherical waves of amplitudes $A_n(\boldsymbol{F})$. The natures of these waves are similar to that given by (20)

Now, recalling the Maxwell's equations

$$\nabla \times \boldsymbol{H} = \sigma\boldsymbol{E} + \in \frac{\partial \boldsymbol{E}}{\partial t}$$

$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$ one can arrive at the following relations:

$$\frac{\partial H_3}{\partial x_2} - \frac{\partial H_2}{\partial x_3} = \sigma E_1 + \in \frac{\partial E_1}{\partial t}$$
$$\frac{\partial H_1}{\partial x_3} - \frac{\partial H_3}{\partial x_1} = \sigma E_2 + \in \frac{\partial E_2}{\partial t}$$
$$\frac{\partial H_2}{\partial x_1} - \frac{\partial H_1}{\partial x_2} = \sigma E_3 + \in \frac{\partial E_3}{\partial t}$$ ...(22)

$$\frac{\partial E_3}{\partial x_2} - \frac{\partial E_2}{\partial x_3} = -\mu \frac{\partial H_1}{\partial t}$$
$$\frac{\partial E_1}{\partial x_3} - \frac{\partial E_3}{\partial x_1} = -\mu \frac{\partial H_2}{\partial t}$$
$$\frac{\partial E_2}{\partial x_1} - \frac{\partial E_1}{\partial x_2} = -\mu \frac{\partial H_3}{\partial t}$$ ...(23)

Replacing $\boldsymbol{F}$ by $\boldsymbol{H}$ and $\boldsymbol{E}$ successively in (21) one can recast (22) and (23) in the forms :

$$\left(\frac{1}{2}\sigma + j\omega \in\right)E_1 = \Phi_2^{H_3}(r,\theta,\phi,t) - \Phi_3^{H_2}(r,\theta,1)e^{-\sigma t/2\in}e^{j\omega t}$$ ...(24)

$$\left(\frac{1}{2}\sigma + j\omega \in\right)E_2 = \Phi_3^{H_1}(r,\theta,1) - \Phi_1^{H_3}(r,\theta,\phi,1)e^{-\sigma t/2\in}e^{j\omega t}$$ ...(25)

$$\left(\frac{1}{2}\sigma + j\omega \in\right)E_2 = \Phi_3^{H_1}(r,\theta,1) - \Phi_1^{H_3}(r,\theta,\phi,1)e^{-\sigma t/2\in}e^{j\omega t}$$ ...(26)

$$\mu\left(\frac{1}{2}\sigma - j\omega \in\right)H_1 = \Phi_2^{E_3}(r,\theta,\phi,1) - \Phi_3^{E_2}(r,\theta,1)e^{-\sigma t/2\in}e^{j\omega t}$$ ...(27)

$$\mu\left(\frac{1}{2}\sigma - j\omega \in\right)H_2 = \Phi_3^{E_1}(r,\theta,1) - \Phi_1^{E_3}(r,\theta,\phi,1)e^{-\sigma t/2\in}e^{j\omega t}$$ ...(28)

and

$$\mu\left(\frac{1}{2}\sigma - j\omega \in\right)H_3 = \Phi_1^{E_2}(r,\theta,\phi,1) - \Phi_2^{E_1}(r,\theta,\phi,1)e^{-\sigma t/2\in}e^{j\omega t}$$ ...(29)

where $\Phi_p^{F_i}(r,\theta,1)$ and $\Phi_p^{F_i}(r,\theta,\phi,1)(\boldsymbol{F} = \boldsymbol{H} \vee \boldsymbol{E})$ are the i^th^ components of the spherical wave function $\left(\frac{\partial \Phi^F}{\partial x_p}\right)(r,\theta)(p = 1,2,3)$ given by

$$\frac{\partial \Phi^F}{\partial x_1}(r,\theta) = \cos\phi \sum_{n\in J^+} A_n(F)\{F_1'(r)P_n(\cos\theta)\sin\theta - F_1(r)P_n'(\cos\theta)(\sin\theta\cos\theta/r)\}$$
$$\frac{\partial \Phi^F}{\partial x_2}(r,\theta) = \cos\phi \sum_{n\in J^+} A_n(F)\{F_1'(r)P_n(\cos\theta)\sin\theta - F_1(r)P_n'(\cos\theta)(\sin\theta\cos\theta/r)\}$$ ...(30)

$$\frac{\partial \Phi^F}{\partial x_3}(r,\theta) = \cos\phi \sum_{n\in J^+} A_n(F)\left\{F_1'(r)P_n(\cos\theta)\cos\theta - F_1(r)P_n'(\cos\theta)\sin^2\theta/r\right\}$$

Hence, one can arrive at the following theorems :

Theorem-1

A longitudinally symmetric electric intensity vector $\boldsymbol{E}$ is said to be associated with time dependent damped spherical wave $\Phi^F(r,\theta,t)$ of frequency w with the damping factor $\sigma/2\in$ iff the bounding surfaces ∂K are conducting ($\sigma \neq 0$) and the components of magnetic intensity vector $\boldsymbol{H}$ are given by (27) to (29) and the frequency w and the wave number *k* are mutually related as $4\in k^2 = \mu(4\in\omega^2 + \sigma^2)$ subject to the restriction $2k\sqrt{\in} > \sqrt{\mu}\,\sigma$.

Theorem2

A longitudinally symmetric magnetic intensity vector $\boldsymbol{H}$ is said to be associated with a time dependent damped spherical wave $\Phi^H(r,\theta,t)$ of frequency w with the damping factor ($\sigma/2\in$) if the bounding surfaces of are conducting ($\sigma \neq 0$) and the components of electric intensity vector are given by (24) to (26) and the frequency w and the wave number *k* are mutually related as $4\in k^2 = \mu(4\in\omega^2 + \sigma^2)$ subject to the restriction $2k\sqrt{\in} > \sqrt{\mu}\,\sigma$.

Determination of current density

A current density consists of displacement current and the conduction current according to Maxwell's theory in electromagnetics[11].

Hence, one can express in the form

$$\boldsymbol{J} = \boldsymbol{J}_c + \boldsymbol{J}_d = \sigma\, \boldsymbol{E}(r,\theta,t) + \in \frac{\partial \boldsymbol{E}}{\partial t}(r,\theta,t) \qquad ...(31)$$

Now, combining the relations (21) and (31), may be finally expressed in the following form

$$\boldsymbol{J} = \Phi^F(r,\theta)e^{-t\left\{\left(\frac{\sigma}{2\in}\right)-J\omega\right\}}\left(\frac{\sigma}{2} + j\omega\in\right) \qquad \left[j = \sqrt{-1}\right] \qquad ...(32)$$

which represents a spherical wave with its modulus and the phase given by the following expressions

$$|\boldsymbol{J}| = \frac{1}{2}\Phi^E(r,\theta)e^{-\sigma t/2\in}\sqrt{\sigma^2 + 4\omega^2\in^2}$$

and phase

$$(\boldsymbol{J}) = \delta + \omega t$$

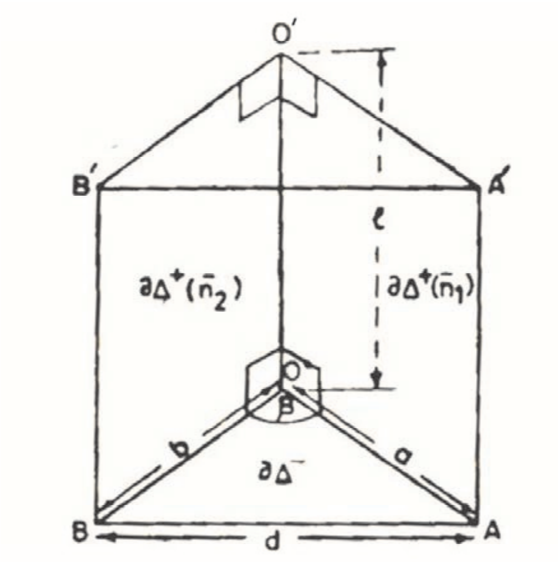

Fig. 1: A convex triangular prism of dimension a, b, d and with flare angle ². OO' is perpendicular to the plane $\Delta' OAB$ and $O'A'B'$

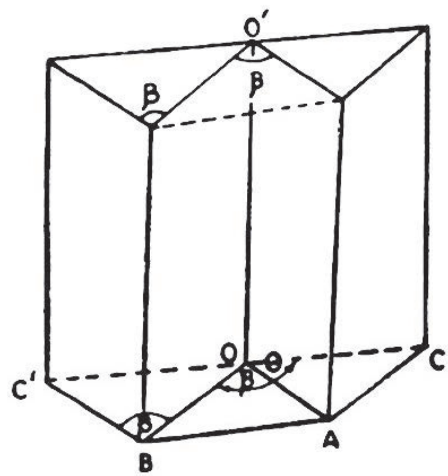

Fig. 2: A model 'M' consists of a triangular prism formed by $\Delta^S OAB$ and $O'A'B'$ and its adjacent groove regions formed by the sides BC' and AC and the sides parallel to OO', OA and OB

$$\tan\delta = \frac{2\omega \in}{\sigma} \quad ...(33)$$

where

CONCLUSIONS

The present paper furnishes the existence of longitudinally symmetric EM waves. Such waves may be designated as longitudinally symmetric spherical waves. The concerning wave functions have been determined as solutions of the governing Maxwell's equations in spherical coordinates. The governing Maxwell's equations have been encountered for finding the magnetic field intensity and electric field intensity vectors. Finally the result has been used for computing the current density.

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