

Estimation of random surface roughness using laser speckle

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ABSTRACT

In this present endeavor, the motivation is to decide the surface roughness and random surface roughness from laser speckle patterns. It may be mentioned that when a monochromatic light is allowed to reflect or transmit from optically rough surface the final intensity pattern at a particular distance is found to be the combination of many coherent components originated from the rough surfaces. The interference of these coherent waves are resulted into laser speckle. Also speckles pattern and their contrasts can be utilized to study the surface roughness. For this purpose the surface is illuminated by plane wave and the image is obtained from the help of lens. A suitable recording medium (air) is utilized for this purpose using Rayleigh scattering formula and scattering cross - section.

Key words: Random surface, coherent components, laser speckle.

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INTRODUCTION

The loss of electromagnetic energy due to the precipitations (Sand, dust,) present in the atmosphere has an important role to create the problem in the utilization of surface roughness by using laser speckle. When stimulated Raman scattering and Brillouin scattering are suppose to study about the study of dusty plasma interactions. It has certain modified permittivity and has been mentioned that the plane of reflected field is simply proportional to the surface height.

The height variations of surface are small compared with the depth of focus of the imaging system. The theory of speckle contrast is confocal microscope. Microscope has been considered by Mendez [1] and he found that for confocal reflecting systems speckle contrast was also minimized in the focal plane. In spite of very effective phenomena very few definitive data regarding the scattering of plane wave from random and rough surfaces are coherent. Therefore in present work an attempt has

been confined to get the roughness due to scattering from random surface. The details of the entire investigations have given in the different sections of the paper.

Basic Theory

Scattering from a Rough Surface

The treatment has developed to measure the scattering from rough surface is based on the kirchoff's approximation[2] and Brillouin formula respectively which are valid for surfaces, where the curvature is large compared with the wave length [3]. We have assume that the height variations of the surface are big, then the expression for image amplitude in a general coherent imaging system can be written as

$$U(r) = \int \int \int_{-\infty}^{\infty} c(m) S(m) \exp[ik(m \cdot r)] d^3 m \quad \dots(1)$$

where, the spatial frequency

$$K_m = k(m_i + n_j + s_k)$$

$c(m)$ is the three - dimensional (3D) Coherent transfer function of the optical System. $S(m)$ is the scattering function of the surface given for a scalar theory by [5].

$$S(m) = \frac{m^2}{2s} \int \int_{-\infty}^{+\infty} \exp[-ik(mx + ny + s\xi(x))] dx dy$$

$$= \frac{m^2}{2s} \int \int_{-\infty}^{+\infty} \delta(z - \xi(x, y)) \exp[-ik(m.r)] d^3r$$

$$= \left(\frac{m^2}{2s} \right) T_s(m) \quad \dots(2)$$

Where $\xi(x, y)$ is the surface profile and $T_s(m)$ is the three-dimensional fourier transform of the surface profile.

Now, Brillouin Scattering treatment has been used to measure the scattering cross-section.

The scattering cross-section is defined as the ratio of scattered power to the incident power thus, the back scattering cross-section of rough surface is given by

$$\sigma_{Bs} = 4\pi r^2 \left(\frac{E_s}{E_i} \right)^2 \quad \dots(3)$$

Where, E_s is the scattered field and E_i is the incident field, then the equation (3) may be modifies as

$$\sigma_{Bs} = \frac{8\pi^3 \mu^2}{3n_R^2 \lambda_0^4} \text{cm}^2 \quad \dots(4)$$

where,

$$n_R = \epsilon_p^{1/2} = \left[1 - \left(\frac{\omega_p}{\omega} \right)^2 \right]^{1/2}$$

Scattering from a Random Surface

The vector potential for A filament or current carrying element at random Surface having roughness can be given as

$$A = \frac{\mu_0 I dl e^{-jK_0 r}}{4\pi r} a_z \quad \dots(5)$$

where A is the vector potential and $I dl$ is the current

element with μ_0 = permeability of the medium in air. Further, for the simplicity of problem the vector potential A can be expressed in terms of components in spherical co-ordinates as

$$a_z = a_r \cos \theta - a_\theta \sin \theta \quad \dots(6)$$

and consequently, putting the value of a_z in equⁿ (6), one gets,

$$A = \frac{\mu_0 I dl}{4\pi r} e^{-jK_0 r} (\alpha_r \cos \theta - \alpha_\theta \sin \theta) \quad \dots(7)$$

Now one has

$$H = \frac{1}{\mu_0} \nabla \times A = \frac{idl \sin \theta}{4\pi} \left(\frac{JK_0}{r} + \frac{1}{r^2} \right) e^{-jK_0 r} a_\phi \quad \dots(8)$$

Where H is the magnetic field and r is relative to the wave length λ_0 then only important terms are thus vary as $1/r$, therefore under such conditions in which radiation field for random Surface is given as

$$E = jZ_0 I dl K_0 \sin \theta \frac{e^{-jK_0 r}}{4\pi r} a_\phi \quad \dots(9)$$

where E is the electric field and $Z_0 = \left(\frac{\mu_0}{\epsilon_0} \right)^{1/2}$

It is evidently, clear that ($a \ll \lambda_0$), the far zone scattered field from the rough surface of the sphere is the same as the radiated by a small electric dipole of total strength P_0 where

$$P_0 = 4\pi a^3 \left(\frac{\epsilon_r - 1}{\epsilon_r + 2} \right) \epsilon_0 E_z a_z \quad \dots(10)$$

where $\epsilon_0 = \epsilon_1 - i\epsilon_2$ is permittivity of the medium in rough surface and ϵ_r is the permittivity of the medium in random surface $j\omega p_0$ As, it is obvious that current element $I dl$ is equivalent to time derivative of dipole moment then may be used to replace in above equation.

thus one has,

$$E_s = -\omega Z_0 K_0 P_0 \sin \theta \frac{e^{-jK_0 r}}{4\pi r} a_0 \quad \dots(11)$$

Since the scattering pattern of a small dielectric sphere is the same as the radiation pattern of a small electric dipole the total scattered power may be obtained as,

$$\begin{aligned} P_s &= \frac{1}{2} Y_0 \int_0^{2\pi} \int_0^\pi |E_s|^2 r^2 \sin \theta d\theta d\phi \\ &= \frac{\omega^2 Y_0 K_0^2 Z_0^2}{12\pi} |P_0|^2 \end{aligned} \quad \dots(12)$$

Combining eqs. (10) and (12), one has

$$P_s = \frac{4}{3} \pi a^2 (K_0 a)^4 Y_0 |E_0|^2 \left| \frac{\epsilon - 1}{\epsilon + 2} \right|^2 \quad \dots(13)$$

The above equation is based on the fact of Rayleigh formula for scattered power for random surface to measure its roughness by scattering approximation.

σ_s . Now, Scattering cross-section is given as

$$\sigma_s = \frac{P_s}{\frac{1}{2} Y_0 |E_0|^2} = \frac{8}{3} \pi a^2 (K_0 a)^4 \left| \frac{\epsilon - 1}{\epsilon + 2} \right|^2 \quad \dots(14)$$

solving

$$\sigma_s = \frac{128}{3} \pi^5 a^2 \left| \frac{\epsilon - 1}{\epsilon + 2} \right|^2 \left(\frac{\alpha}{\lambda_0} \right)^4 \quad \dots(15)$$

This expression for absorption cross-section, one can be obtained as from equation (12)

$$\begin{aligned} P_s &= \frac{1}{2} Y_0 \int_0^{2\pi} \int_0^\pi |E_s|^2 r^2 \sin \theta d\theta d\phi \\ &= \frac{\omega^2 Y_0 K_0^2 Z_0^2}{12\pi} |P_0|^2 \end{aligned}$$

Hence, absorption cross section is obtained by dividing P_s by power incident density (P_i), one has

$$\begin{aligned} \sigma_a &= \frac{P_s}{P_i} = \frac{\omega^2 Y_0 K_0^2 Z_0^2}{P_i} |P_0|^2 = \frac{\omega^2 Y_0 K_0^2 Z_0^2 |P_0|^2}{\frac{1}{2} Y_0 |E_0|^2} \\ &= 2 \omega^2 K_0^2 Z_0^2 \left| \frac{P_0}{E_0} \right|^2 \end{aligned}$$

The expression for extinction cross section can be utilized to measure the quantity of attenuation of the signal and scattering co-efficient of propagation in random surface.

EXPERIMENTAL

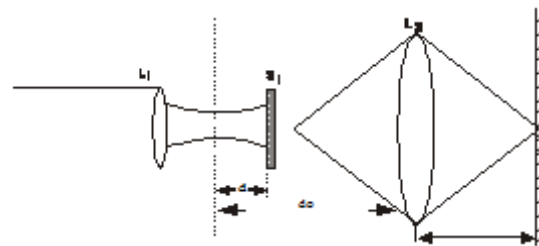


Fig. 1

Geometry for laser speckle pattern in random surface

We measure the scattering pattern of a small dielectric sphere which is considered as same with radiation pattern of a small electric dipole. Since it has been assumed that ($a \ll \lambda_0$), so far-zone scattered field remains same compared with radiated by a small electric dipole moment in a random surface.

RESULTS AND DISCUSSION

One can distinguish coarse and fine surfaces using the eqs.(9) and (12) respectively, similarly the rough and smooth surface can be easily known using eqs. (15) and (16). The total scattering power is useful to study about the lateral speckle pattern on the basis of its radiated for zone scattering field. We can use the equation (16) as the expression to study about extinction cross section for different precipitation particles like (dust particle, raindrop and snow particle) to calculate as a function of frequency and particle density. The equation (16) and (18) are in good agreement with Rayleigh scattering theory. The radius of the

precipitation and roughness of random surface as well rough surface is very much less than the incident wave. At higher frequency the radius of the particle may be compared to the free space wave length and back scattering cross section approaches to its geometrical cross-section. Hence, the concept of Rayleigh scattering as well as dipole radiation can be used to predict the roughness in laser speckle patterns. The nature of the speckles and

distance between the speckles at random surface have been plotted in form of graphs 1(a) and 1(b) at different angles. We can distinguish between rough and smooth surface on the one hand, on the other hand we distinguish between coarse and fine surfaces. For a smooth surface, the surface can be all in focus setting and the problem reduces to 2D one.

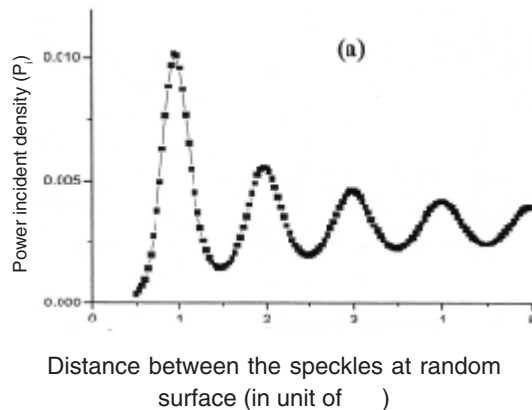


Fig. - 2(a)

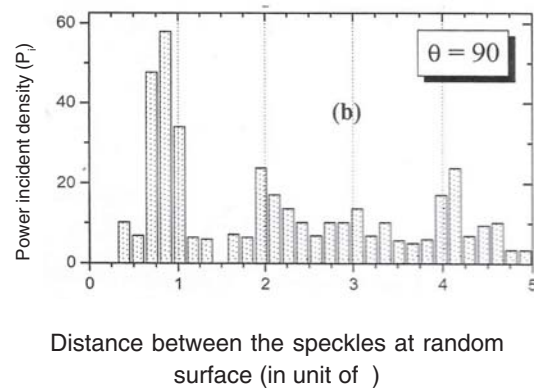


Fig. - 2(b)

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$$\theta = 45^\circ$$

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