

VISCOUS FLOW THROUGH A POROUS MEDIUM PAST AN OSCILLATING PLATE IN A ROTATING SYSTEM WITH MAGNETIC EFFECTS

Ch. V. Ramana Murthy and S. B. Kulkarni

Department of Applied Sciences,
Finolex Academy of Management & Technology, Ratnagiri - 415 639 (India)

(Received, 12 August, 2005)

ABSTRACT

Three dimensional unsteady state flow of an incompressible viscous fluid in the presence of transverse magnetic field through an oscillating porous plate in a rotating system, when the whole system being in the state of solid body rotation with constant angular velocity about the z-axis normal to the plate is discussed. The cases of uniform suction and injection (blowing) on the boundary has been considered. The effects of the magnetic parameter, porous medium, the suction and injection (blowing) on the velocity field, skin friction and temperature distribution has been shown graphically. It is found that the velocity field can be controlled by proper selection of porosity factor and or magnetic field parameter in both cases of suction/injection.

Keywords: Viscous flow of fluid, Oscillating plate and magnetic field parameter.

INTRODUCTION

The study of flows through porous media has been motivated by its immense importance and continuing interest in several fields of science, engineering, technology, bio physics, astro physics and space dynamics. The applications are many in medical field. For economic use, an appropriate process design requires accurate flow entities on the bounding surface. The oscillatory flows are of frequent occurrence in nature having several applications mainly in all chemical and nuclear reactors. Therefore, due to increasing importance in technological and physical problems, the topic had received the attention of several researchers.

The solution describing the hydrodynamic boundary layer flow in a rotating system with uniform suction and injection had been investigated by Kishore *et al*¹. Later, Gupta² obtained an exact solution of the three dimensional Navier Stokes equations under the steady state condition for the flow past a plate with uniform suction/injection in a rotating system while Debnath and Mukherjee³ studied the motion of an in compressible, homogeneous, viscous fluid bounded by a porous plate with uniform suction/injection. Subsequently, Puri⁴ examined the case of fluctuating flow of a viscous fluid on a porous plate in a rotating

medium. The magnetic field effect on the free convection and mass transfer flow through porous medium with constant suction and heat flux has been examined by Acharya *et al*⁵. Thereafter, Kim⁶ investigated the case of unsteady state two dimensional laminar flow of a viscous incompressible electrically conducting polar fluid through a porous medium in the presence of a transverse magnetic field. Recently, Jat *et al*⁷ studied the problem of MHD viscous flow through a porous medium past an oscillating plate in a rotating system by employing the method of Laplace transforms to examine the effect of various flow parameters encountered in the problem.

In the present analysis, an attempt has been made to study the three dimensional unsteady flow of an incompressible viscous fluid in the presence of transverse magnetic field through a porous medium past an oscillating plate subject to uniform suction and injection in a rotating system. Instead of employing the method of Laplace transforms, the method of separation of variables has been employed to examine the nature of field entities.

Formulation of the problem

In the present situation, the whole of the system is considered to be in a state of solid body rotation with constant angular velocity Ω about

the z-axis normal to the plate and in addition to it, a non torsional oscillation of a given frequency is imposed on the plate in its own plane at time $t > 0$ for the generation of unsteady flow in rotating system. The equations of motion describing the unsteady flow of a viscous incompressible and electrically conducting fluid through a porous medium bounded by a porous plate in the presence of an external uniform magnetic field in a rotating coordinating system in the standard description are:

$$\text{div } \mathbf{u} = 0 \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + 2\Omega \hat{\mathbf{K}} \times \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} - \frac{\nu}{K} \mathbf{u} - m\mathbf{u} \quad (2)$$

where $m = \frac{\sigma_c B_0^2}{\rho}$ (magnetic parameter)

$$\rho \mathbf{u} \cdot \nabla (c_p \mathbf{T}) = \nabla \cdot (k \nabla \mathbf{T}) \quad (3)$$

where

$\mathbf{u} = (u, v, w)$ is the velocity vector is the unit vector along the z-axis, p the pressure including the centrifugal term, the density and the kinematic viscosity and K the permeability of the medium. Throughout the analysis, it is assumed

that is constant, where w_0 is the suction or injection parameter according as $w_0 > 0$ or $w_0 < 0$ and the velocity field depends on z and so that

$$\mathbf{u}(z, t) = [u(z, t), v(z, t), -w_0].$$

The equation of continuity is identically satisfied. In the absence of the pressure gradient, eqn (2) can be written as :

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} - \frac{\nu}{K} u - mu \quad (4)$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\nu}{K} v - mv \quad (5)$$

while of equation for energy can be written as:

$$-w_0 \frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial z^2} \quad (6)$$

The initial and boundary conditions are:

$$t < 0: u = v = 0, T = T_w \quad z \quad (7)$$

$$t \geq 0: u = U e^{i\omega t}, v = 0, w = w_0 \text{ and } T = T_w \text{ at } z = 0 \quad (8)$$

$$u = v = 0 \text{ and } T = T_\infty \text{ as } z \rightarrow \infty \quad (9)$$

Solution of the problem

Letting $q = u + iv$, the governing equations of motion can be merged into a single equation as:

$$\frac{\partial q}{\partial t} - w_0 \frac{\partial q}{\partial z} = \nu \frac{\partial^2 q}{\partial z^2} - \frac{\nu}{K} q - m q \quad (10)$$

For the purpose of realistic situation, attention is being paid only to the real component of q . Introducing the following non dimensionalization scheme:

$$z^* = \frac{zU}{\nu}, \quad t^* = \Omega t, \quad K^* = \frac{KU^2}{\nu^2}, \quad q^* = \frac{q}{U}, \quad E = \frac{2\Omega\nu}{U^2} \quad (11)$$

$$P_r = \frac{\mu C_p}{\kappa}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad S = \frac{w_0}{U}, \quad \gamma = \frac{\omega}{\Omega} \quad (12)$$

the governing equation of motion in the non dimensional form will now be:

$$\frac{\partial q^*}{\partial t^*} - S \frac{\partial q^*}{\partial z^*} = \frac{\partial^2 q^*}{\partial z^{*2}} - K^* q^* - P_r q^* \quad (13)$$

where (magnetic interaction parameter) and the energy Eqn yields:

$$\frac{\partial T^*}{\partial t^*} - S \frac{\partial T^*}{\partial z^*} = \frac{\partial^2 T^*}{\partial z^{*2}} \quad (14)$$

The non - dimensional form of the initial and boundary conditions are now

$$t \leq 0: q^* = 0, T^* = 1 \quad \forall z^* \quad (15)$$

$$t > 0: q^* = e^{bt}, T^* = 1 \quad \text{at } z^* = 0 \quad (16)$$

$$q^* \rightarrow 0 \text{ or finite and } T^* = 0 \text{ as } z^* \rightarrow \infty \quad (17)$$

Instead, of employing the Laplace transform technique as used by Jat *et al*⁷, the method of separation of variables has been employed in the present investigation. Letting $q^* = X(z^*)Y(t)$, eqn (13) together with the initial and boundary conditions will reduce to:

$$X'' + \lambda^2 X = 0 \quad (18)$$

together with the conditions:

$$\text{as } z^* \rightarrow \infty, q^* = 0 \quad (19)$$

$$\text{and at } z^* = 0, q^* = \exp(bt) \quad (20)$$

The solution of eqn (13) satisfying the initial and boundary conditions is given by:

$$q^* = \exp(bt) \cos(\lambda z^*) \quad (21)$$

while for that of the energy is given by:

$$T^* = \exp(bt) \quad (22)$$

where

Deductions

1. In the absence of magnetic field i.e., $M = 0$ we conclude that the velocity distribution achieved by Thronley⁸ can be obtained as and .
2. The results obtained are in agreement with that of Gupta² if $M = 0$ and as $M \rightarrow \infty$.
3. The results obtained by Debnath and Mukerjee³ can be realized as $M \rightarrow \infty$.

RESULTS AND DISCUSSION

The present analysis describes the characteristic features of the electrically conducting viscous fluid flow through a porous medium in the presence of a transverse magnetic field in a rotating system with uniform suction and injection respectively. The whole system is considered to be in a state of solid body rotation with constant angular velocity about the z - axis normal to the plate. The solution is obtained by considering the porous plate oscillating with a given frequency about the mean velocity and constant magnetic field. Figs 1,2,3 illustrates the effect of the magnetic field, permeability of the medium and the angular velocity with which the entire system is rotated for the case of uniform suction on the velocity field, while Figs 4,5 and 6 projects the effect of magnetic field and permeability of the medium and the angular velocity with which the entire system is rotated for the case of injection.

1. It is observed from Fig - 1 that, for a given permeability of the medium and suction on the flow, the increase in the magnetic parameter results in the decrease in the velocity of the fluid in the boundary layer region.
2. Fig - 2 illustrates the effect of porosity of the medium on the velocity profiles. It is noticed that in the case of constant magnetic parameter, as the porosity increases, the velocity in the boundary layer region increases.
3. The nature of the velocity profiles as the entire system is subjected to the rotation for constant values of the magnetic parameter and porosity has been illustrated in Fig - 3. It is observed that as the angular velocity increases, the velocity of the fluid draining over the plate decreases due to the centrifugal force and possibly the fluid slides down the plate well within the boundary layer region.
4. Fig - 4 shows the effect of magnetic parameter on the fluid velocity. It is observed that for the constant porosity factor while maintaining constant fluid injection, as the magnetic parameter increases, there is a decrease in the fluid velocity in the boundary region. Further, upon merging of Fig - 1 with Fig - 4, it is seen that the effect of injection is to drive away the velocity profiles from the bounding surface which is in agreement with the reality.

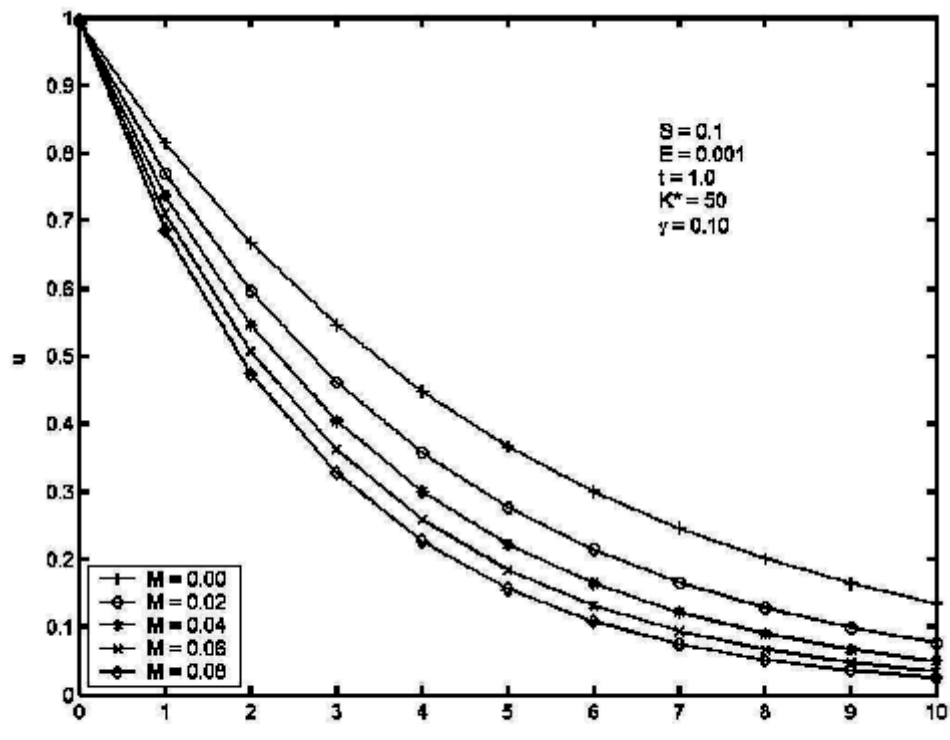


Fig - 1: Effect of magnetic parameter on the velocity profiles for the case of suction

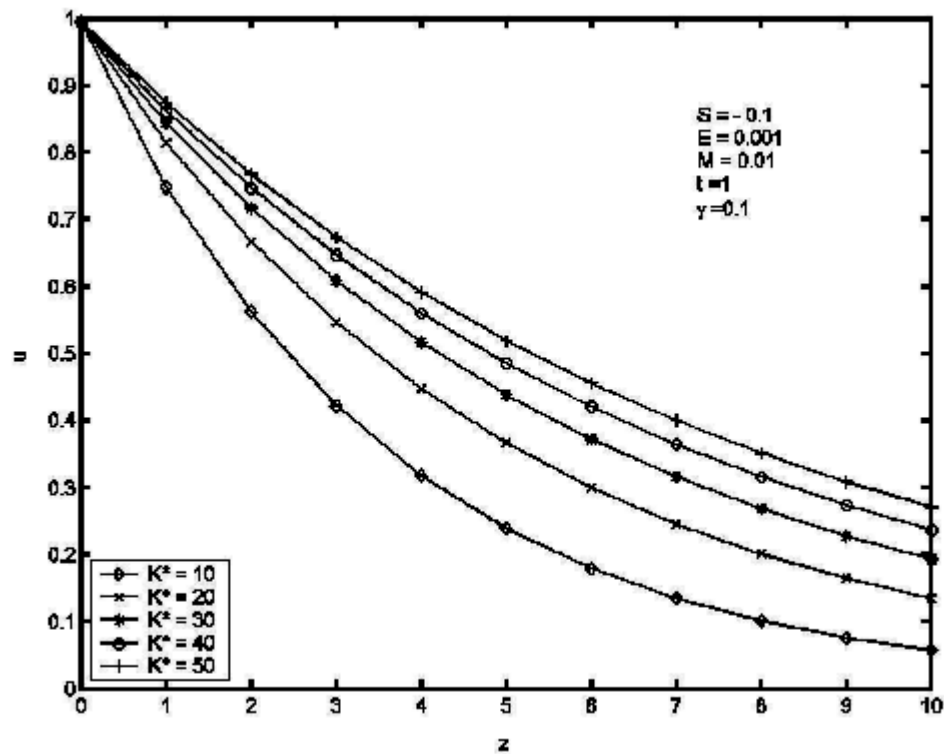


Fig - 2: Effect of porosity parameter on the velocity profiles for the case of suction

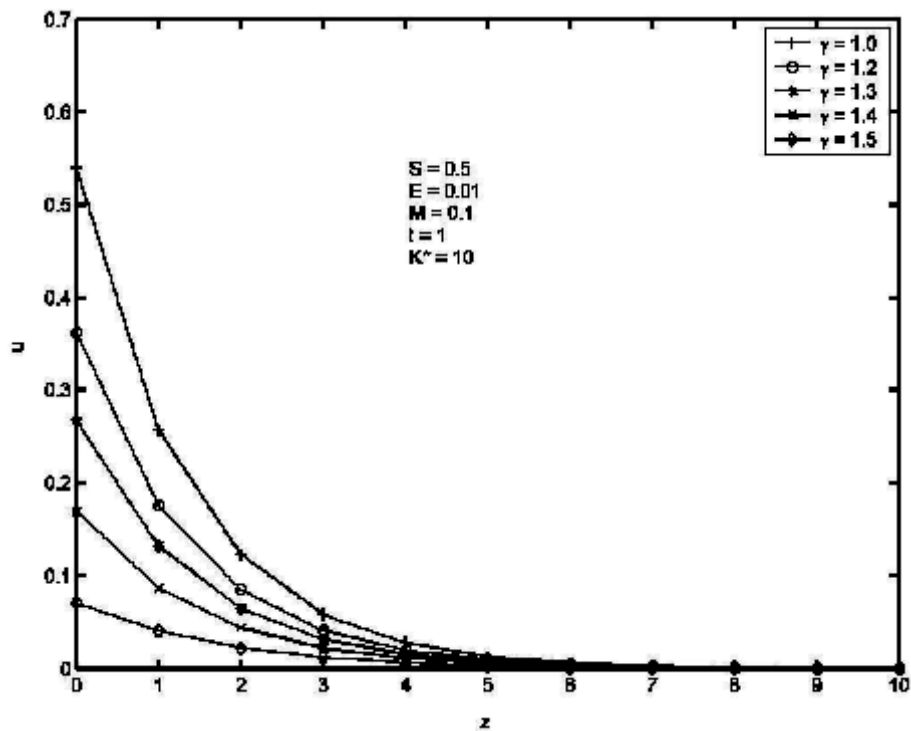


Fig - 3: Effect of angular velocity parameter on the velocity profiles for the case of suction

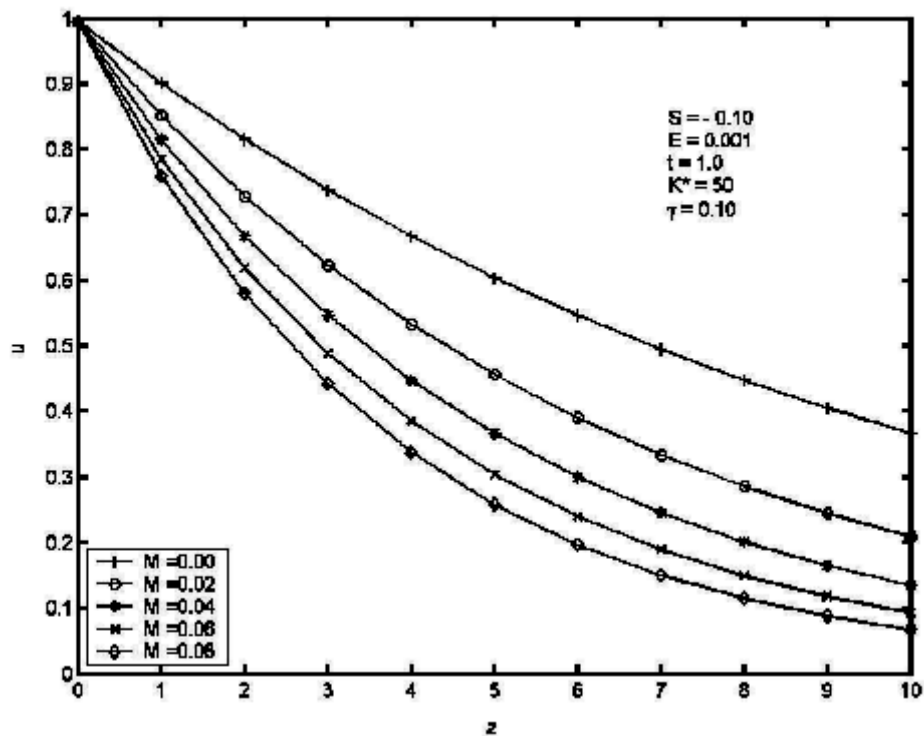


Fig - 4: Effect of magnetic parameter on the velocity profiles for the case of injection

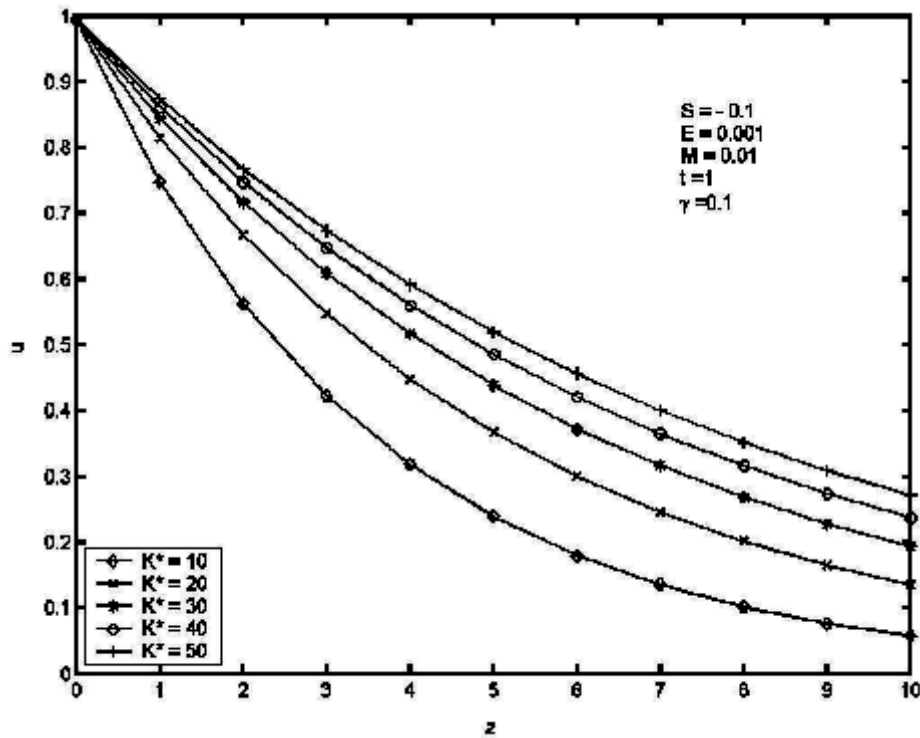


Fig - 5: Effect of porosity parameter on the velocity profiles for the case of injection

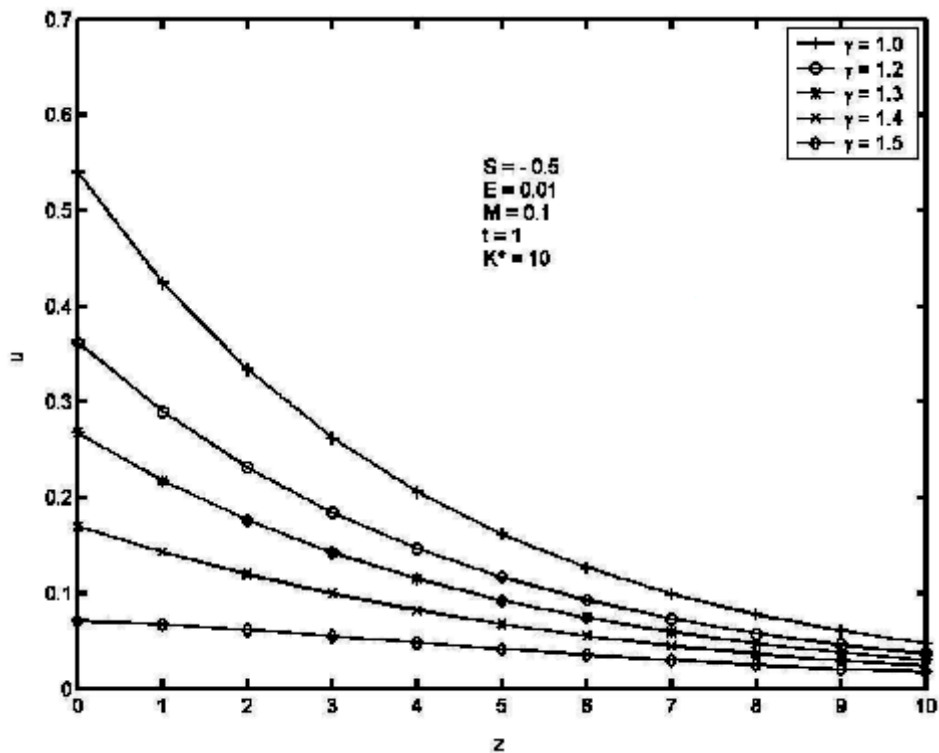


Fig - 6: Effect of angular velocity parameter on the velocity profiles for the case of injection

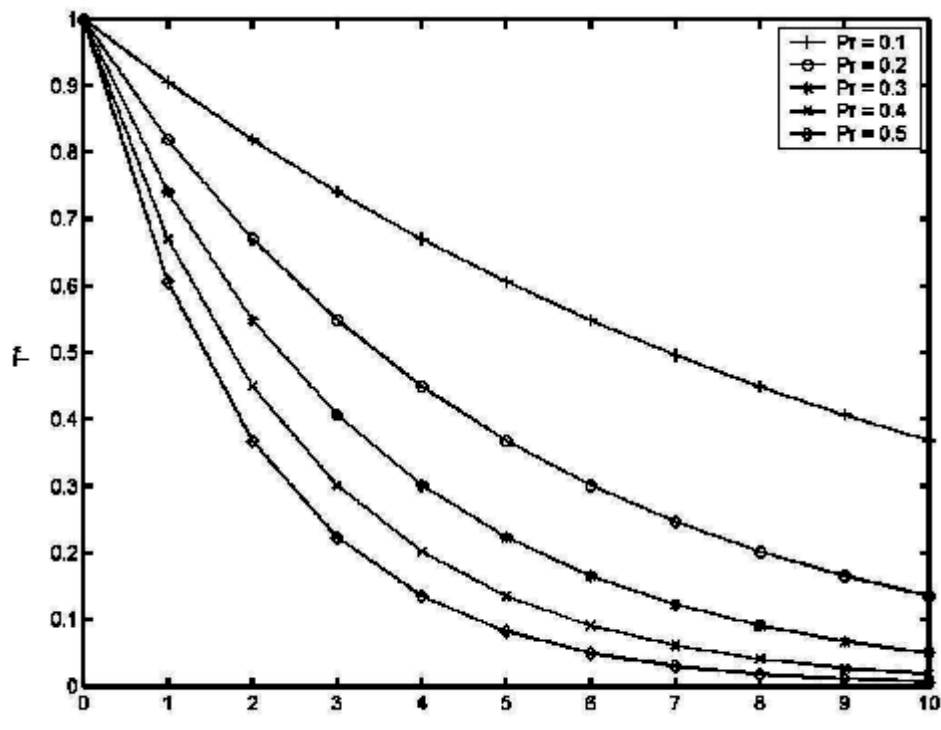


Fig - 7: Effect of Prandtel no. on the temperature distribution

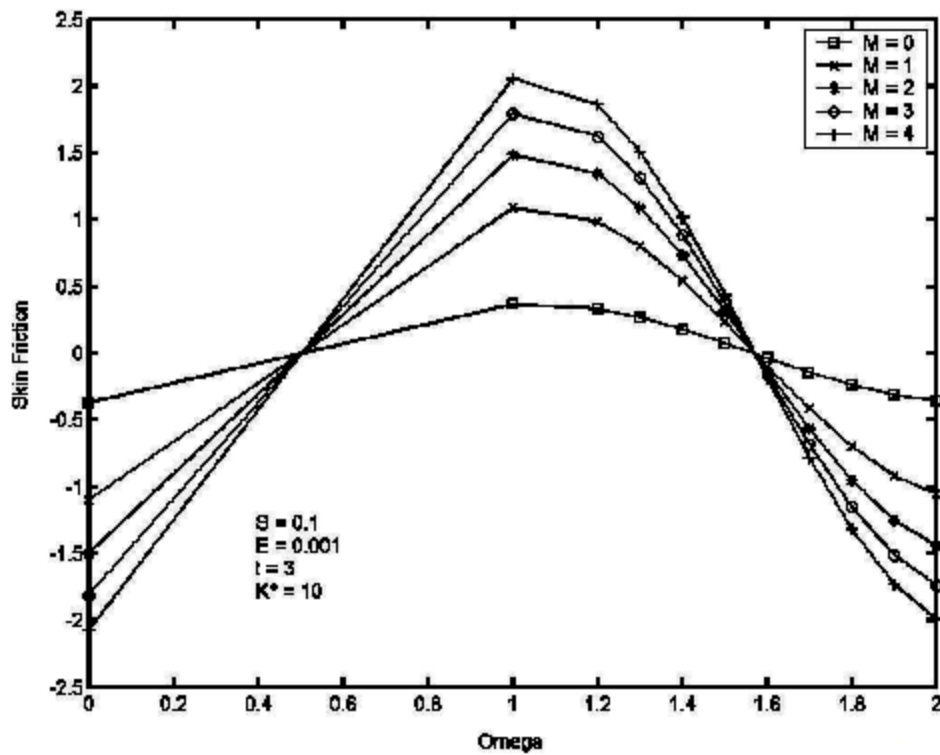


Fig - 8: Effect of magnetic parameter on the skin friction for the case of suction

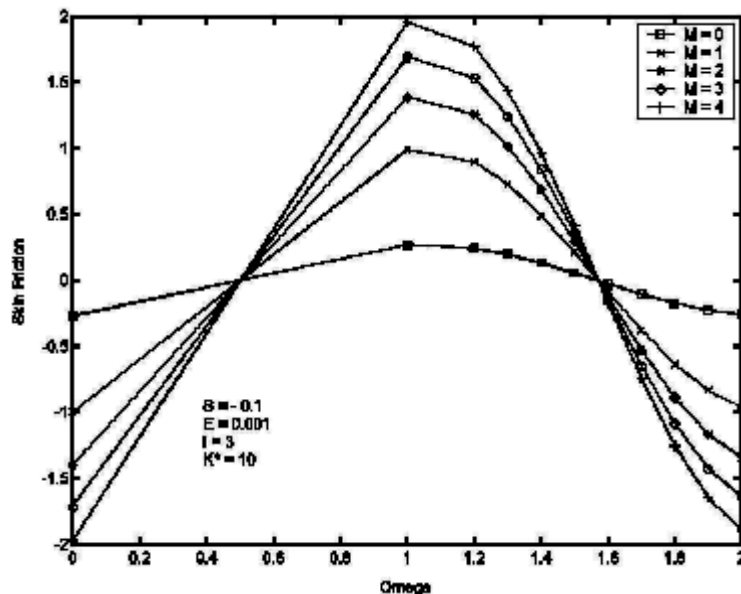


Fig - 9: Effect of magnetic parameter on the skin friction for the case of injection

5. The distribution of the velocity profiles in case of constant injection, as the porosity of the plate increases in the case of constant magnetic parameter is illustrated in Fig - 5. It is observed that as the pore size of the medium increases, the fluid velocity increases in the core boundary region. This effect can be attributed due to the fact that the injection from the medium is responsible for the increase in the velocity of the fluid in the boundary layer region. Further, upon merging the Fig - 2 with Fig - 5, it is seen the effect of suction is to pull back the velocity profiles towards the plate.
6. The nature of the velocity profiles as the entire system is subjected to the rotation for constant values of the magnetic parameter and porosity with the effect of injection has been illustrated in Fig - 6. It is noticed that as the angular velocity increases, the velocity of the fluid draining over the plate decreases due to the centrifugal force and possibly the

fluid slides down the plate well within the boundary layer region.

7. It is seen from Fig - 7 that, as the Prandtl number increases, there is decrease in the temperature field.
8. Fig - 8 and Fig - 9 illustrates the effect of magnetic parameter on skin friction when a constant suction is applied on the boundary. The skin friction on the plate increases with the increase of magnetic influence. Further, it is seen that the effect of injection of fluid on the boundary is to increase the skin friction on the bounding surface.

Conclusions

From the above facts, it can be concluded that, the velocity field increases with the increase of suction and injection as well as permeability of the medium and decreases with the increase of magnetic parameter. Hence, the velocity field can be controlled by proper selection of porosity or magnetic field in either case of suction/injection.

REFERENCES

1. Kishore N, Tejpal S & Tiwari R A, *Indian J Pure Appl Math*, **12** (1), 111 (1968)
2. Gupta A S, *Phys Fluids*, **15** (5), 930 (1972)
3. Debnath L & Mukerjee S, *Phys fluids*, **16** (9), 1418 (1973)
4. Puri P, *Acta Mech*, **21**, 53 (1975)
5. Acharya M, Dass G C & Singh L P, *Indian J Pure Appl Math*, **31**(1), 1 (2000)
6. Kim Y J, *Int J Heat Mass Transfer*, **44**, 2791 (2001)
7. Jat R N & Anuj K Jhankal, *Indian J Eng and Mat Sciences*, **10**, 37 (2003)
8. Thronely C, *J Appl Math*, **21**, 451 (1968)